SLOW OR NO HEATING IN MANY-BODY QUANTUM SYSTEMS

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joint work with

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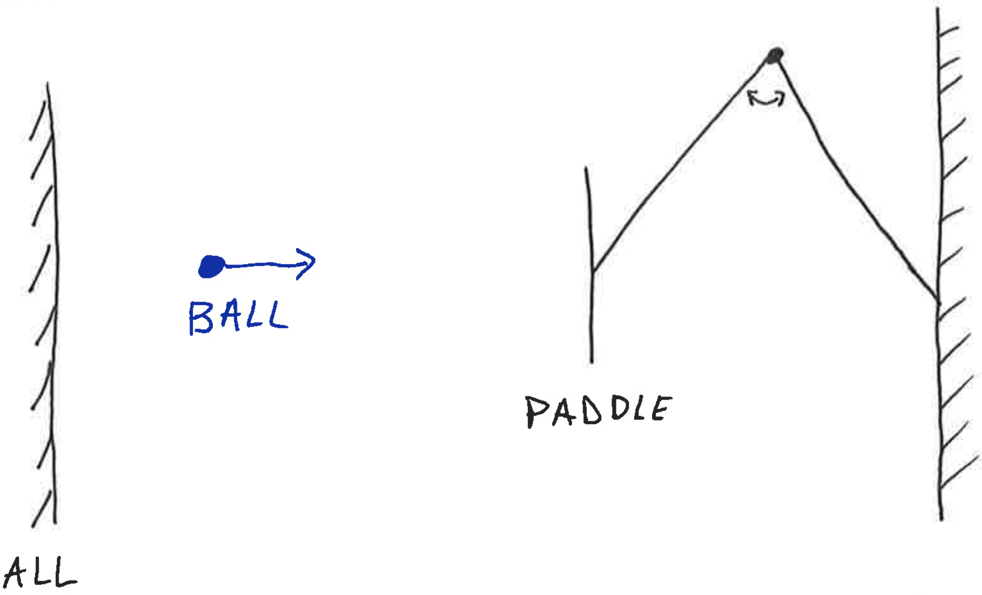
Aim: TO DESCRIBE QUANTUM MANY-BODY SYSTEMS WITH BREAKDOWN OR QUASI-BREAKDOWN (i.E. FOR A VERY LONG TRANSIENT TIME) OF ERGODICITY.

- MOSTLY DRIVEN SYSTEMS
- EMERGENCE OF (QUASI)-CONSERVED QUANTITIES
- "PRETHERMALIZATION"

EXAMPLE: PING-PONG



ELASTIC COLLISIONS WITH THE WALL AND PADDLE (SINGE PARTICLE CLASSICAL PROBLEM)

TWO EXTREME CASES:

(a) THE PADDLE MOVES RANDOMLY:

ERGODIC

$\Rightarrow$ THE BALL ABSORBS ENERGY FOREVER

$$E(t) \sim t.$$

$\Rightarrow$ "HEATING UP TO INFINITE TEMPERATURE"

(b) THE PADDLE MOVES PERIODICALLY

LOCALIZED

$\Rightarrow$ THE VELOCITY OF THE BALL STAYS BOUNDED (IF DRIVING IS SMOOTH)

$$|v(t)| \leq C \qquad \forall t.$$

$\Rightarrow$ EMERGENCE OF AN EFFECTIVE CONSERVED QUANTITY.

(SEE DOLGOPYAT, DE SIMOI)

QUESTION: CAN WE FIND A SIMILAR DICHOTOMY IN MANY-BODY QUANTUM SYSTEMS ?

PLAN OF THE TALK:

- BASIC INTUITION ON THIS PROBLEM (FOLLOWING D'ALESSIO – POLKOVNIKOV '12 AND SUBSEQUENT CRITICISM)
- PARTICULAR CASE OF MANY-BODY LOCALIZED (MBL) SYSTEMS
- DRIVEN ERGODIC SYSTEMS WITH QUASI-CONSERVED QUANTITIES
- GENERALIZATION TO NON-DRIVEN SYSTEMS.

MAIN MATHEMATICAL TOOL:

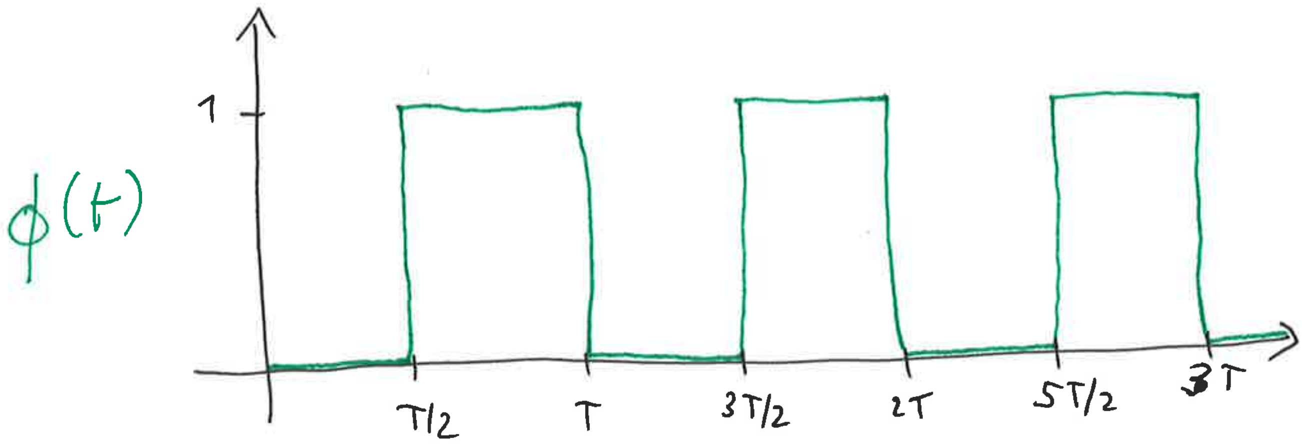
SUCCESSIVE CANONICAL TRANSFORMATIONS FOR MANY-BODY SYSTEMS (CFR. IMBRIE '14)

HIGH FREQUENCY PERIODIC DRIVING

HAMILTONIAN:

$$H(t) = H^{(0)} + g\, V(t), \qquad V(t+T) = V(t)$$

e.g. $V(t) = \phi(t)\, \bar{V}$ WITH $\phi(t)$ LIKE



(SWITCH BETWEEN TWO HAMILTONIANS)

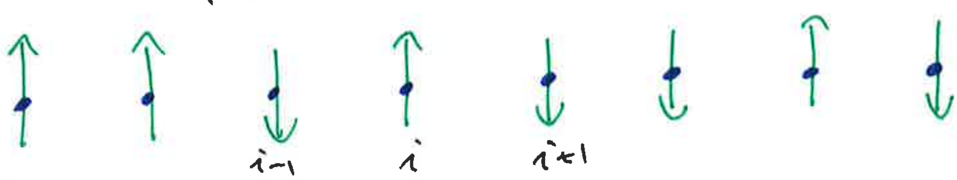
$H^{(0)}$ AND $\bar{V}$ ARE SUMS OF LOCAL TERMS:

$$H^{(0)} = \sum_{i\in\Lambda} H_i^{(0)}, \qquad \bar{V} = \sum_{i\in\Lambda} \bar{V}_i$$

e.g. 1-d SPIN CHAIN

$$H^{(0)} = \sum_{i\in\Lambda} h_i\, \sigma_i^{(z)} + J_i^{(x)}\, \sigma_i^{(x)}\, \sigma_{i+1}^{(x)}$$

$$\bar{V} = \sum_{i\in\Lambda} J_i^{(z)}\, \sigma_i^{(z)}\, \sigma_{i+1}^{(z)}$$



$$i \in \Lambda, \qquad \Lambda \longrightarrow \infty$$

HOW TO DEFINE HEATING UP TO $T=\infty$?

VIA FLOQUET THEORY :

- UNITARY DYNAMICS : $|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$
- DEFINE AN EFFECTIVE HAMILTONIAN:

$$\boxed{U(T) = e^{-iH_{eff}T}}.$$

(ALWAYS POSSIBLE AT $|\Lambda| < +\infty$)

2 CASES AS $\Lambda \longrightarrow \infty$

(a) H_{eff} is A SUM OF LOCAL TERMS (OR QUASI-LOCAL TERMS) :

LOCALIZED

$$H_{eff} = \sum_{i\in\Lambda} H_{eff,i}$$

$\Rightarrow$ BEHAVES WELL AS $\Lambda \to \infty$

$\Rightarrow$ EFFECTIVE CONSERVED QUANTITY

$\Rightarrow$ BOUND ON HEATING UP.

(b) H_{eff} is NOT A SUM OF LOCAL TERMS

$\Rightarrow$ NO LIMIT AS $\Lambda \longrightarrow \infty$

$\Rightarrow$ SYSTEM GOES TOWARDS INFINITE TEMPERATURE STATE = MAXIMAL ENTROPY.

ERGODIC

MAGNUS / BHC EXPANSION

SETTLE THE ISSUE VIA AN EXPICIT EXPANSION ?

$$e^{A} e^{B} = e^{A+B+\frac{1}{2}[A,B]+\dots}$$

HERE:

$$e^{-i\frac{T}{2}H^{(0)}} \cdot e^{-i\frac{T}{2}(H^{(0)}+g\overline{V})} = e^{-iTH_{eff}}$$

$$\Rightarrow \quad H_{eff} = \left(H^{(0)} + \frac{g}{2}\overline{V}\right) + T\,\frac{ig}{4}[H^{(0)},V] + \mathcal{O}(T^2)$$

NAIVE HOPE: $\quad \tilde{E}$: ENERGY SCALE

(a) $T\tilde{E} \ll 1$ (HIGH FREQUENCY):

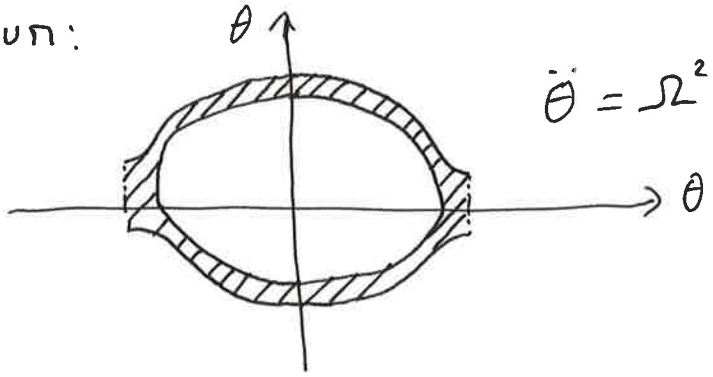
EXPANSION CONVERGES

(b) $T\tilde{E} \gtrsim 1$ (LOW FREQUENCY):

EXPANSION DIVERGES

REMARK:

CONSISTENT WITH INTUITION FROM FORCED PENDULUM:



$$\ddot\theta = \Omega^2 \sin\theta + g\sin(\theta-\omega t)$$

$$|\text{STOCHASTIC LAYER}| \sim g\, e^{-\frac{|\omega|}{\Omega}}$$

FREE SYSTEMS $\left(\text{SAY} \quad H(t) = \Gamma(h(t))\right)$

IN THIS CASE, THIS IS CORRECT!

WHY?

ENERGY ABSORPTION REMAINS BOUNDED IF

$$\boxed{\omega = \frac{2\pi}{T} > W}$$

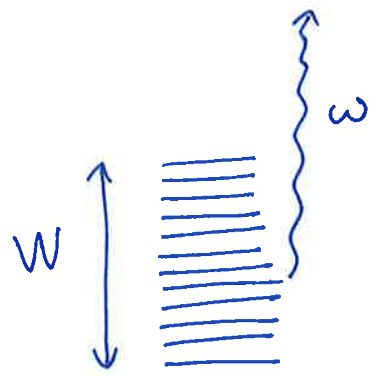
FREQUENCY

SINGLE PARTICLE BANDWIDTH

ENERGY ABSORPTION RATE:

$$\Gamma^{ii}_{\beta}(\omega) \sim g^2 \sum_{\eta,\eta'} e^{-\beta E_\eta} \left|\langle \eta' | \bar{V}_i | \eta \rangle\right|^2 \delta(E_{\eta'} - E_\eta - \omega)$$

(LINEAR RESPONSE IN g = FERMI GOLDEN RULE)



"SYSTEM CANNOT ABSORB A PHOTON WITH FREQUENCY $\omega > W$".

WHAT ABOUT MORE GENERIC SYSTEMS?

INTERACTING SYSTEMS

IN GENERAL, ONE EXPECTS BHC EXPANSION TO BE ONLY ASYMPTOTIC AT SMALL T.

WHY?

LET US CONSIDER AGAIN

$$\Gamma_\beta^{ii}(\omega) \sim g^2 \sum_{\eta,\eta'} e^{-\beta E_\eta} |\langle\eta|V_i|\eta'\rangle|^2 \, \delta(E_\eta - E_{\eta'} - \omega)$$

(NOW $|\eta\rangle, |\eta'\rangle$ ARE MANY-BODY EIGENSTATES!)

TRICK:

$$\langle\eta'|V_i|\eta\rangle = \frac{\langle\eta'|[H,V_i]|\eta\rangle}{E_{\eta'}-E_\eta}$$

$$= \cdots = \frac{\langle\eta'|[H,[\cdots,[H,V_i]\cdots]|\eta\rangle}{(E_{\eta'}-E_\eta)^n}$$

THEN:

$$\| [H,[\cdots,[H,V_i]\cdots] \| \leq \varepsilon^n n!$$

SO: FOR $E_{\eta'} - E_\eta = \omega$ and $\frac{\omega}{\varepsilon} \gg 1$:

$$\int_{D\omega} d\omega \, \Gamma_\beta^{ii}(\omega) \sim \frac{n!}{(\omega/\varepsilon)^n} \underset{\uparrow}{\sim} \boxed{e^{-\omega/\varepsilon}}$$

OPTIMIZATION: $n \sim \frac{\omega}{\varepsilon}$

REMARK 1

FOR SYSTEMS SATISFYING EIGENSTATE THERMALIZATION HYPOTHESIS (ETH), ONE EXPECTS NO BETTER BOUND:

$$|\langle \eta' | V | \eta \rangle|^2 \sim \frac{f(E_{\eta'} - E_\eta)}{\dim(\mathcal{H})}$$

REMARK 2

IN $d=1$, ARAKI ANALITICITY ALLOWS TO IMPROVE THE EXPONENTIAL BOUND INTO SOMETHING SUPER-EXPONENTIAL

SEE OUR PAPER PRL 115 FOR MORE DETAILS.

LET US NOW LOOK FOR A BETTER TECHNOLOGY THAN BHC!

ROTATING FRAME

UNITARY EVOLUTION FOR GENERAL TIME t:

$$U(t) = \underbrace{P(t)}_{\text{PERIODIC AND UNITARY}} e^{-i H_{eff} t}$$

PERIODIC: $P(t+T) = P(t)$
AND UNITARY

$P(t)$: . ROTATING FRAME TRANSFORMATION

. SOLVES THE EQUATION:

$$\boxed{P^{+}(t) \left(H(t) - i \frac{d}{dt} \right) P(t) = H_{eff}}$$

BETTER WAY TO GET H_{eff}

- ANALOGY WITH TIME-INDEPENDENT SET-UP: FIND U s.t. $U^{+} H U =$ DIAGONAL. (SEE IMBRIE '14)
- HERE: FIND $P(t)$ PERIODIC s.t. H_{eff} is
  - TIME INDEPENDENT
  - TIME INDEPENDENT UP TO VERY SMALL CORRECTIONS

WE NOW SEE THREE EXAMPLES:

I. QUASI-CONSERVED EXTENSIVE QUANTITIES

- REMIND LINEAR RESPONSE PREDICTION:

$$\Gamma(\omega) \sim e^{-\omega/\varepsilon}, \qquad \omega > 0,$$

FOR GENERAL LATTICE HAMILTONIANS WITH BOUNDED ON-SITE ENERGY (ε)

- WE WANT TO EXTEND THIS RESULT BEYOND LINEAR RESPONSE REGIME ($|g|>0$)

- WE TAKE

$$H = H^{(0)} + g\,\phi(\omega t)\cdot V$$

$$g \ll \varepsilon, \qquad g \ll \omega, \qquad \frac{1}{T}\int_0^T \phi(\omega s)\,ds = 0$$

ENERGY PER SITE IN $H^{(0)}$.

- TAKE $P(t)$ OF THE FORM

$$P(t) = e^{\frac{g}{\omega}A(\omega t)}, \qquad A(\omega t) = \sum_i A_i(\omega t)$$

IN

$$P^{\dagger}(t)\left\{ H(t) - i\frac{d}{dt}\right\} P(t) \approx H_{eff}$$

WE WILL NOT REACH EQUALITY!

IMPORTANT: SUCH $P(t)$ PRESERVES QUASI-LOCALITY: IF O_i IS LOCAL ON SITE $i \in \Lambda$, THEN $P^+(t)\, O_i\, P(t)$ IS QUASI-LOCAL AROUND SITE $i \in \Lambda$:

$$P^+(t)\, O_i\, P(t) = \sum_{B \ni i} \tilde{O}_B$$

WITH $\| \tilde{O}_B \| \leq \left(\frac{g}{\omega}\right)^{|B|}$.

- HOW TO SELECT $A(\omega t)$? EXPAND IN $\frac{g}{\omega}$ AND IMPOSE THAT THE HIGHEST ORDER TERM THAT DEPENDS ON t VANISHES:

$$P^+(t) \left\{ H(t) - i \frac{d}{dt} \right\} P(t)$$

$$= H^{(0)} + \underbrace{g\, \phi(\omega t)\, \bar{V}} + \mathcal{O}\left(\frac{g}{\omega} \cdot \varepsilon\right)$$

$$\underbrace{- i g \frac{dA}{dt}(\omega t)} + \mathcal{O}\left(\frac{g}{\omega} \cdot g\right)$$

CANCEL THIS

$$\Longrightarrow \quad A(\omega t) = \left(\int_0^{\omega t} ds\, \phi(s) \right) \cdot \bar{V}$$

PERIODIC SINCE $\frac{1}{T} \int_0^T ds\, \phi(\omega s) = 0$.

- UPSHOT: IN THE ROTATING FRAME, WE HAVE A NEW HAMILTONIAN WITH SMALLER TIME-DEPENDENT PART:

$$g \longrightarrow \max\left\{\frac{g}{\omega}\cdot\varepsilon,\ \frac{g}{\omega}\cdot g\right\} \ll g$$

- WE ITERATE THIS PROCEDURE n TIMES
  - × WE GAIN POWERS OF g/ω $\Longrightarrow$

$$g \xrightarrow{n} \max\left\{\left(\frac{g}{\omega}\right)^n\varepsilon,\ \left(\frac{g}{\omega}\right)^n g\right\}$$

  - × WE HAVE TO STOP AT SOME OPTIMAL n AS TERMS BECOME LESS AND LESS LOCAL

RESULT:

WE OBTAIN A QUASI-CONSERVED QUANTITY UP TO VERY LONG (QUASI-) EXPONENTIAL TIMES IN ω/ε. WE CALL IT $\tilde{H}_{eff}$:

$$\boxed{\frac{1}{|\Lambda|}\left\| U^{+}(kT)\,\tilde{H}_{eff}\,U(kT) - \tilde{H}_{eff}\right\| \leq C\cdot(kT)\cdot e^{-\frac{\omega}{\varepsilon}\frac{1}{1+\log\frac{\omega}{\varepsilon}}}}$$

$$\boxed{\tilde{H}_{eff} = \sum_i \tilde{H}_{eff,i}}$$

II. HUBBARD MODEL (NO DRIVING)

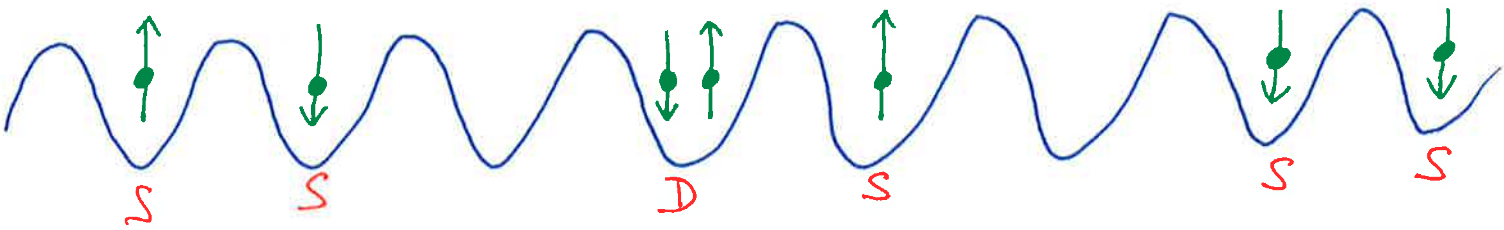
FERMIONS ON THE LATTICE WITH

$$H = \underbrace{U \sum_{i} n_{i\uparrow} n_{i\downarrow}}_{\text{INTERACTION}} + \underbrace{t \cdot \sum_{i\sim j, \sigma} a_{i,\sigma} a^{+}_{j,\sigma}}_{\text{HOPPING}}$$

WE CONSIDER THE REGIME

$$\frac{U}{t} \gg 1 .$$

PARTICLES COME IN SINGLONS / DOUBLONS:



VERY HARD TO CREATE / DESTROY DOUBLON:
BIG ENERGY MISMATCH !

$$\Delta E = U \gg t .$$

TRUE CONSERVED QUANTITIES:

- ENERGY
- NUMBER OF FERMIONS WITH SPIN $\uparrow$
- NUMBER OF FERMIONS WITH SPIN $\downarrow$
- NUMBER OF

QUASI-CONSERVED QUANTITY:

DRESSED VERSION OF NUMBER OF DOUBLONS

FOR EXPONENTIAL TIMES IN U/t.

$\tilde{N}_D$ IS SUCH THAT $\tilde{N}_D = \sum_i \tilde{N}_{D,i}$

AND

$$\frac{1}{|\Lambda|} \| U^{+}(t)\, \tilde{N}_D\, U(t) - \tilde{N}_D \| \leq C \cdot t \cdot e^{-\frac{U}{t} \cdot \frac{1}{1+\ln \frac{U}{t}}}$$

$t =$ HOPPING PARAMETER

HERE!

WE ADAPT OUR PREVIOUS METHOD:

- WE WRITE:

$$H = U N_D + t H_{pre} + t H_{des}$$

NUMBER OF DOUBLONS

HOPPING PRESERVING N_D

HOPPING CREATING OR DESTROYING DOUBLONS

- TRY TO GET RID OF H_{des} VIA CANONICAL TRANSFORMATIONS THAT PRESERVE LOCALITY:

TAKE $\quad P = e^{\frac{t}{U} A}, \qquad A = \sum_i A_i$

THEN

$$P^\dagger H P = H + \frac{t}{U}[H, A] + \left(\frac{t}{U}\right)^2 [A, [A, H]] + \dots$$

$$= U . N_D + t H_{pure} + t H_{des}$$

$$+ t [N_D, A] + O\left(\frac{t}{U} \cdot t\right)$$

SELECT A TO CANCEL :

$$\Rightarrow \quad [A, N_D] = H_{des}$$

$$\Rightarrow \quad \langle \eta' | A | \eta \rangle = \frac{\langle \eta' | H_{des} | \eta \rangle}{N_D(\eta) - N_D(\eta')}$$

ALWAYS $\neq 0$ WHEN $\langle \eta' | H_{des} | \eta \rangle \neq 0$

$|\eta\rangle$: CLASSICAL CONFIGURATIONS

! $A = \sum A_i$ BECAUSE $H_{des} = \sum_i H_{des, i}$

- ITERATE THIS PROCEDURE

III. MBL SYSTEMS (WITH DRIVING)

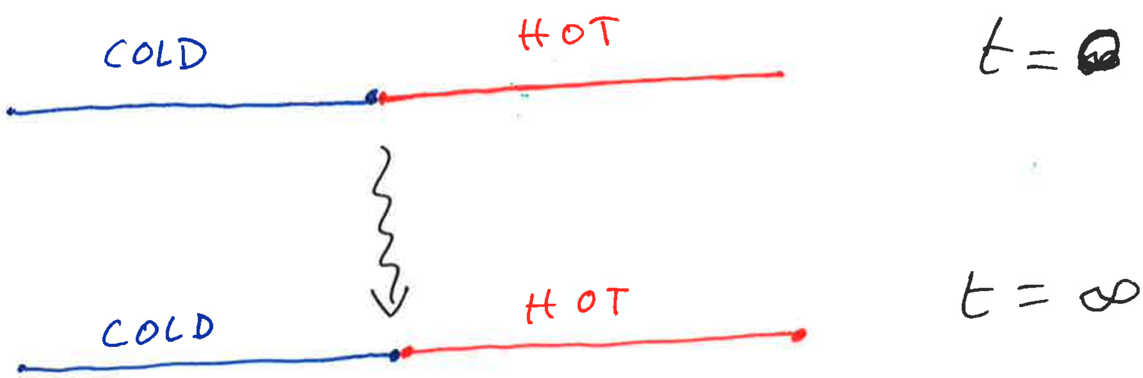
MANY-BODY LOCALIZED (MBL) SYSTEMS HAVE EIGENSTATES THAT DO NOT OBEY ETH

$\Longrightarrow$ SOME PREVIOUS ARGUMENT DOES NOT APPLY

$\Longrightarrow$ WHAT IS THE BEHAVIOR OF MBL SYSTEM UNDER DRIVING?

VERY SHORT INTRO TO MBL:

- ANALOGOUS OF ANDERSON LOCALISATION FOR INTERACTING MANY-BODY SYSTEMS
- COMPLETE LACK OF THERMALIZATION:



- TYPICAL EXAMPLE: 1d SPIN CHAIN

$$H = \sum_i \left\{ h_i \sigma_i^{(z)} + J^{(z)} \sigma_i^{(z)} \sigma_{i+1}^{(z)} + J^{(x)} \sigma_i^{(x)} \right\}$$

- DEFINITION VIA EIGEN STATES

UNITARY U S.T. $U^{+} H U = \Delta$ (DIAGONAL)

$\boxed{\text{" } H \text{ is MBL} \iff U \text{ PRESERVES LOCALITY "}}$

PRESERVING LOCALITY MEANS

O_i LOCAL AT $i \in \Lambda$ $\Rightarrow$

$$U^{+} O_i U = \sum_{A \ni i} \tilde{O}_A , \quad \| O_A \| \leq C \cdot e^{-\frac{|A|}{\xi}}$$

WITH ξ : LOCALIZATION LENGTH

- LOCAL INTEGRALS OF MOTION (ABANIN, HUSE, OGANESYAN ...)

$$\Delta = \sum_i c_i \sigma_i^z + \sum_{i<j} c_{i,j} \sigma_i^z \sigma_j^z + \sum_{i<j<k} (\cdots) + \cdots$$

$$\boxed{\tau_i^z = U \sigma_i^z U^{+} \qquad \text{(LIOMs)}}$$

$$H = \sum_i c_i \tau_i^z + \sum_{i<j} c_{i,j} \tau_i^z \tau_j^z + \sum_{i<j<k} (\cdots) + \cdots$$

$$| c_{i_1, \ldots, i_m} | \leq C e^{-\frac{i_m - i_1}{\xi}} .$$

$\Rightarrow$ DYNAMICS = COLLECTION OF INDEPENDENT QUASI-LOCAL PSEUDO-SPINS (τ_i^z)

DRIVEN MBL SYSTEMS

$$H(t) = \underset{\text{MBL}}{H^{(0)}} + g\, V(t)$$

MBL

$V(t+T) = V(t)$ AND
$\frac{1}{T}\int_0^T ds\, V(s) = 0$.

SAME QUESTION AS BEFORE: CAN WE

FIND $H_{eff} = \sum_i H_{eff,i}$ S.T.

$$P^\dagger(t) \left\{ H(t) - i \frac{d}{dt} \right\} P(t) = H_{eff} \;?$$

PERHAPS YES! WHY?

DUE TO LIOMS STRUCTURE, IT IS NO LONGER TRUE THAT THE SYSTEM CAN ABSORB ANY FREQUENCY BY A LOCAL MOVE: MBL MATERIAL CANNOT ACT AS BATH!

RESULT (NON-MATHEMATICAL)

H_{eff} EXISTS AND IS ITSELF MBL IF

$$\frac{g}{\omega} \ll 1 \,, \qquad \frac{g^2}{\omega W} \ll 1$$

W: DISORDER STRENGTH

REMARK 1

EXAMPLE OF BREAKDOWN OF LINEAR RESPONSE:

MOTT LAW (OR ANALOGOUS MANY-BODY COUNTER PART) PREDICTS

$$\sigma(\omega) \sim \omega^2$$

NON-EQUILIBRIUM PHENOMENON

- TRANSIENT REGIME WHERE LR APPLIES: SYSTEM HEATS UP A BIT
- NEXT, SYSTEM SATURATES AND REACHES NON-EQUILIBRIUM STATE: LR NOT VALID ANYMORE

REMARK 2

OUR CONDITIONS ALLOW $g \gg W$

$\Rightarrow$ FOR MOST INDIVIDUAL TIMES, $H(t)$ MAY NOT BE MBL

REMARK 3

HOW TO GET THE RESULT ?

CANONICAL TRANSFORMATIONS AGAIN!

$$P^{+}(t)\left\{H(t)-i\frac{d}{dt}\right\}P(t)=H_{eff}$$

DIFFERENCE WITH PREVIOUS

- $H^{(0)}$ IS LOCAL IN EIGENSTATE BASIS
- $\Rightarrow$ ALLOWS TO CANCEL MORE TERMS AT ONCE
- $\Rightarrow$ QUADRATIC CONVERGENCE (À LA KAM)
- $\Rightarrow$ ALLOWS TO OVERCOME THE PROBLEM OF TOO MANY COMMUTATORS.