Interacting bosons on a ring with a gauge field

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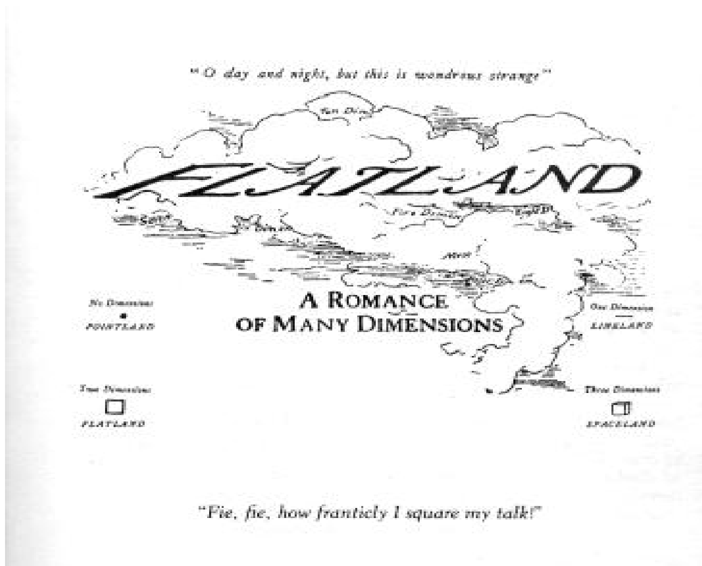
A tour in « lineland »



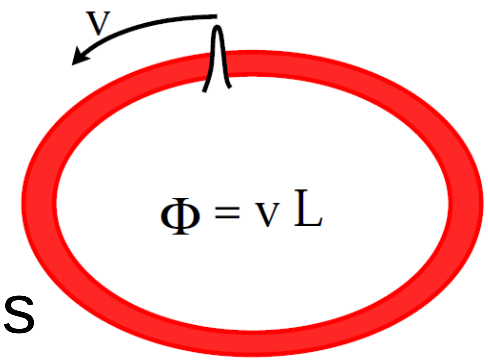
- Worse than 'Flatland'
- One dimensional systems are very peculiar : from the traffic jam problem to the Tonks classical model for hard rods

- Particles cannot circumvent each other
- *Increased effect of interactions*

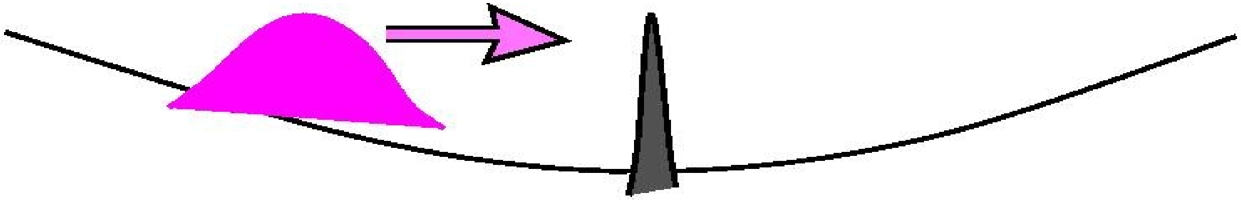
Plan

- One-dimensional quantum gases : experimental realizations and essential facts

- Interacting bosons on a ring:
  – persistent currents
  – macroscopic superpositions of current states



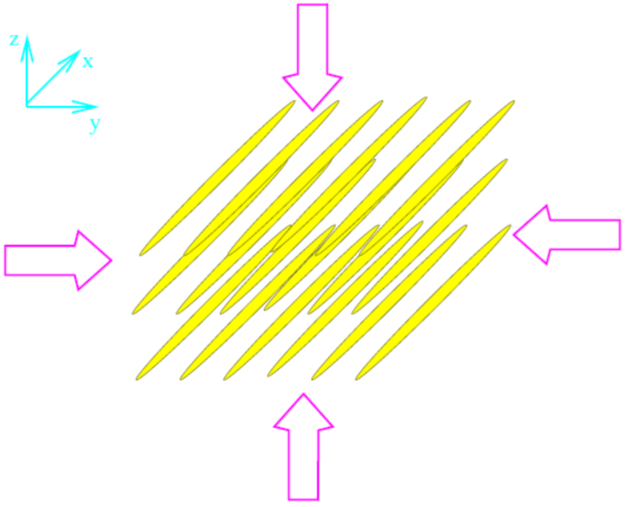
- Interacting bosons in harmonic trap
  – dipole modes in a split trap



One-dimensional quantum systems : definition

- Cylindrical geometry
- Very large transverse confinement

Realization : 2D optical lattices



- All energy scales smaller than transverse energy

$$\mu, k_B T \ll \hbar\omega_\perp$$

Interactions in 1D quantum gases

- Interactions due to atom-atom collisions

  (short range, s-wave scattering length)

- Effective 1D interactions

$$v(x) = g\delta(x)$$

$$g = 2a_s\hbar\omega_\perp(1 - 0.4602\, a_s/a_\perp)^{-1}$$

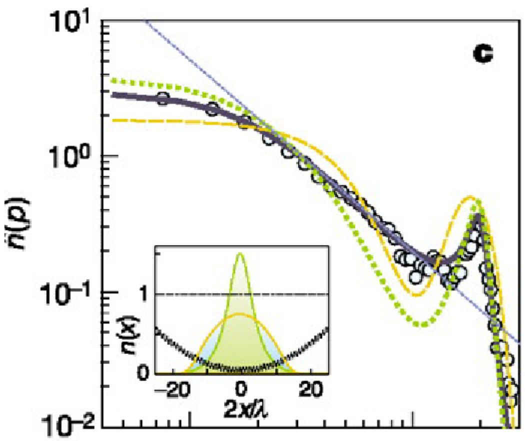
- Hamiltonian (Lieb-Liniger)

$$\mathcal{H} = \sum_i -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(x_i) + g\sum_{i<j}\delta(x_i - x_j)$$

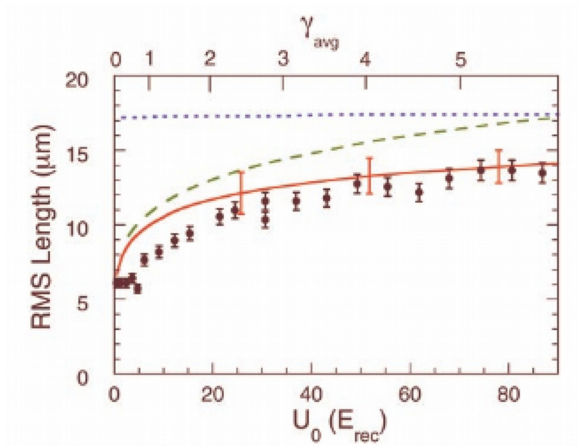
- Dimensionless interaction strength :

$\gamma = gn/(\hbar^2 n^2/m)$ interaction energy/kinetic energy

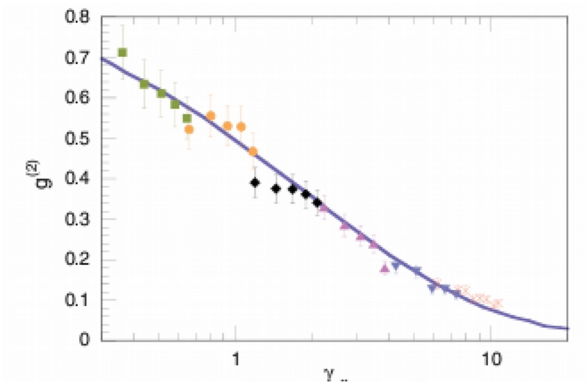
One-dimensional quantum gases : experiments



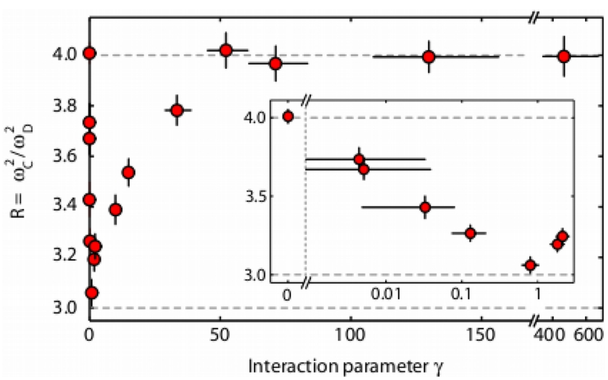
B. Paredes et al (2004)



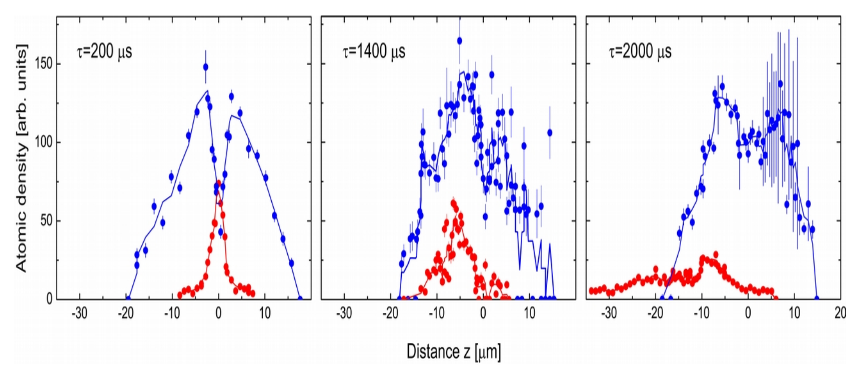
T. Kinoshita et al (2004)



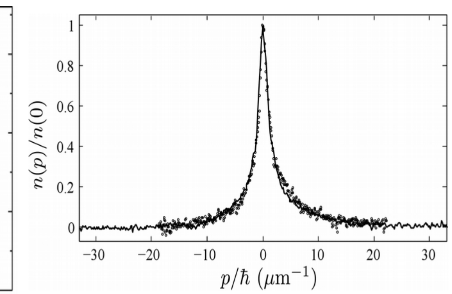
T. Kinoshita et al (2005)



E. Haller et al (2009)



S. Palzer et al (2009)



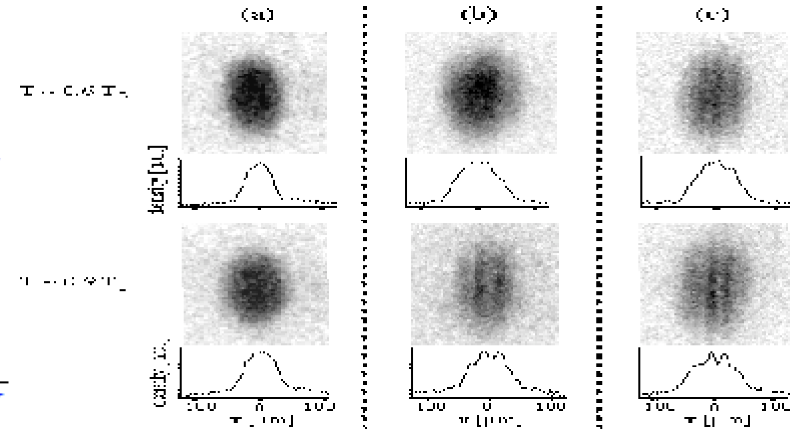
T. Jacqmin et al (2011)

Strongly interacting regime reached in the experiment

Important quantum fluctuations in 1D

- No Bose-Einstein condensation in uniform 1D system – phase fluctuations increase at increasing interactions

$$\rho_1(x,x') = \langle \Psi^\dagger(x)\Psi(x')\rangle \to \frac{1}{|x-x'|^{1/2K}}$$

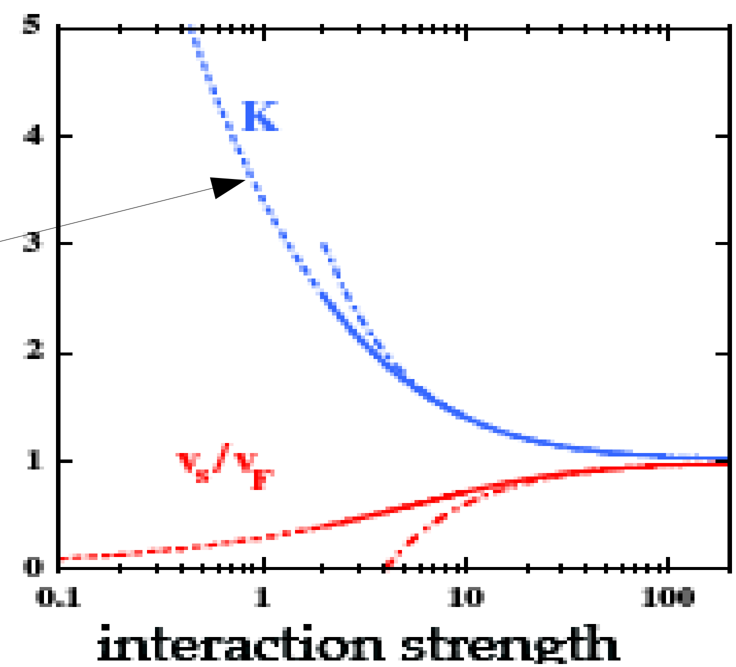


Hannover, 2001

- Duality : density fluctuations decrease at increasing interactions

$$\langle \rho(x)\rho(0)\rangle \sim |x|^{-2K}$$

K = Luttinger parameter,
depending on interaction strength



M. Cazalilla, JPhysB 2001

- Strongly interacting TG regime reached in experiments with ultracold gases

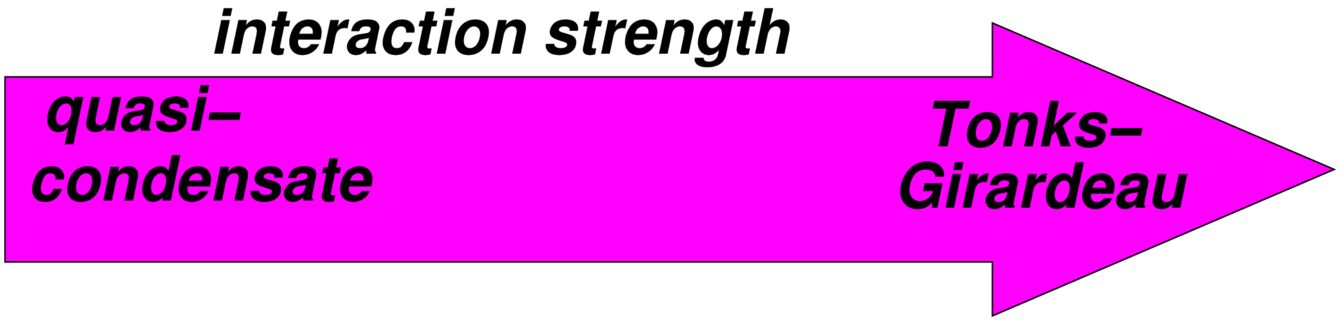
Line diagram for 1D bosons

- Weak interactions:

  a condensate with fluctuating phase

- Strong interactions:

  fermionization



- No phase transition in uniform wires
- But very different behaviour from weak to strong interactions

The weakly interacting regime : Gross-Pitaevskii equation

- Mean-field description of a Bose gas

  with 'condensate wavefunction' $\Phi(x,t)$

$$i\hbar\partial_t\Phi = -\frac{\hbar^2\nabla^2}{2m}\Phi + V_{ext}\Phi + g|\Phi|^2\Phi$$

External confinement　　　interactions

- Nonlinear Schroedinger equation: superfluidity, solitons...
- Even in the dilute regime Eint/Ekin<<1, under external confinement/disorder the interactions are important *[Baym-Pethick, Stringari]*

Intermediate interactions : Luttinger liquid theory

- Quantum hydrodynamics, low-energy theory

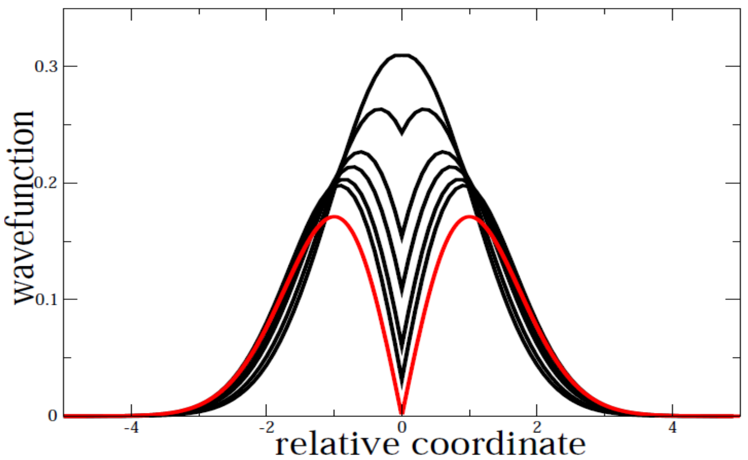
  for the *superfluid phase* ϕ

  and the *density fluctuation* $\partial_x \theta$

$$\mathcal{H} = \frac{\hbar v_s}{2\pi} \int dx K (\partial_x \phi)^2 + \frac{1}{K} (\partial_x \theta)^2$$

- Sound velocity and Luttinger parameter (→compressibility) from the microscopic theory
- Phonon excitation spectrum: valid at intermediate and large interactions

Strong interactions : the Tonks-Girardeau gas



- Infinitely strong repulsions mimick Pauli principle
- Exact solution *[Girardeau, 1960]* mapping onto a Fermi gas

$$\Psi_B(x_1, ..., x_N) = \mathcal{A}\,\det[\phi_j(x_\ell)]$$

with

$$\mathcal{A} = \Pi_{1\leq j\leq \ell\leq N}\mathrm{sign}(x_j - x_\ell)$$

….also valid for:

- inhomogeneous systems
- finite-temperature properties
- out-of-equilibrium dynamics

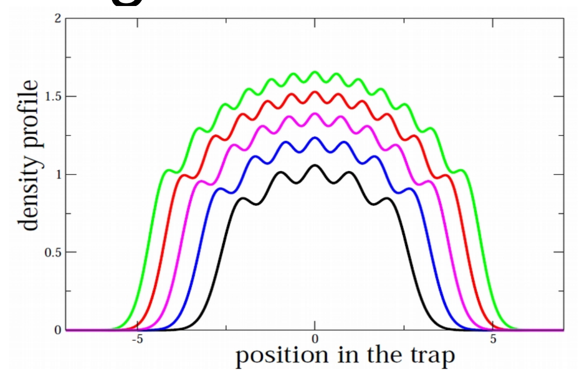
No length scale associated to interactions : scale invariance

Fermionic properties

- The density profile coincides with the one of a Fermi gas

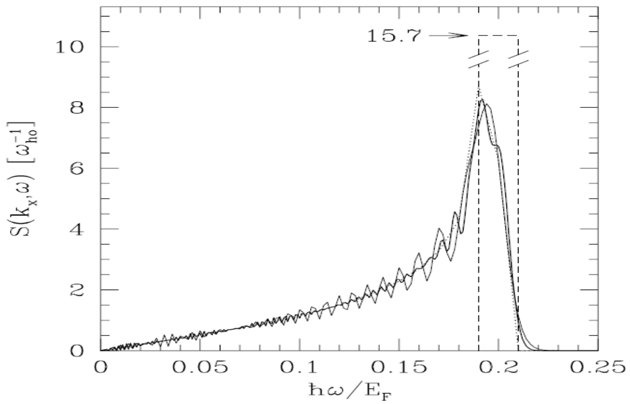
- Also density-density correlations are fermionic

- *Green's function method for large N*



[P Vignolo, AM, MP Tosi (2000)]

- *Dynamic structure factor*



[P Vignolo, AM, MP Tosi (2001)]

Bosonic properties

- One-body density matrix

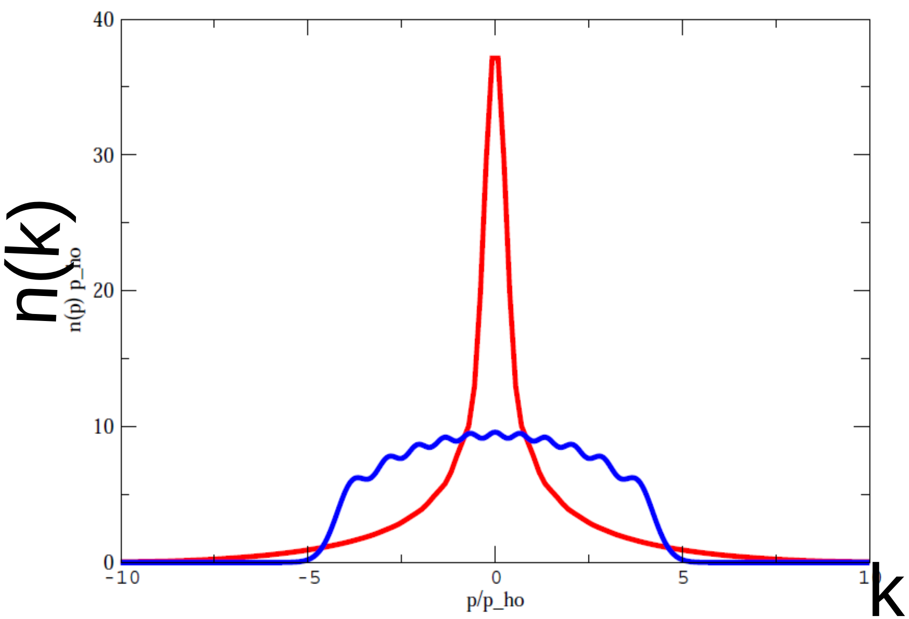
$$\rho_1(x,y) = \langle \Psi^\dagger(x)\Psi(y)\rangle$$

- *not a Bose-Einstein condensate* :
  $\sqrt{N}$ occupancy of |k=0>

- Momentum distribution

$$n(k) = \int dxdy\, e^{ik(x-y)}\rho_1(x,y)$$

in harmonic trap :

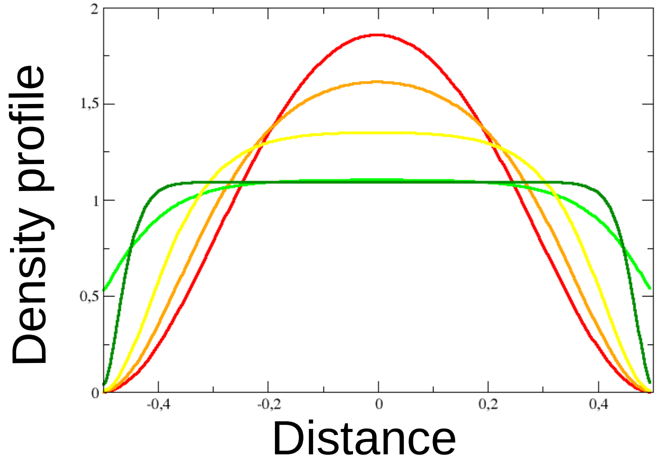


bosonic vs fermionic

(Technical point : analytical simplifications reduce a many-body integration to simpler form)

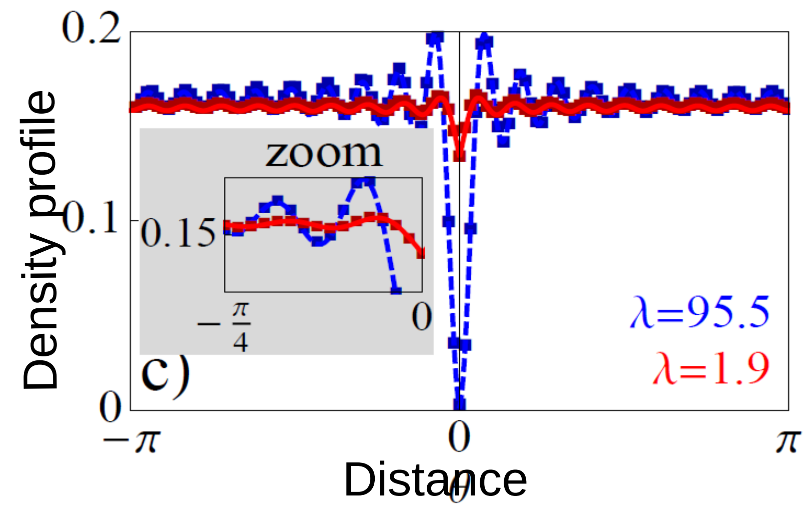
1D interacting Bose gas *with a barrier*

- An interacting fluid adjusts in proximity of a barrier :



– healing of a Bose-Einstein condensate near a wall

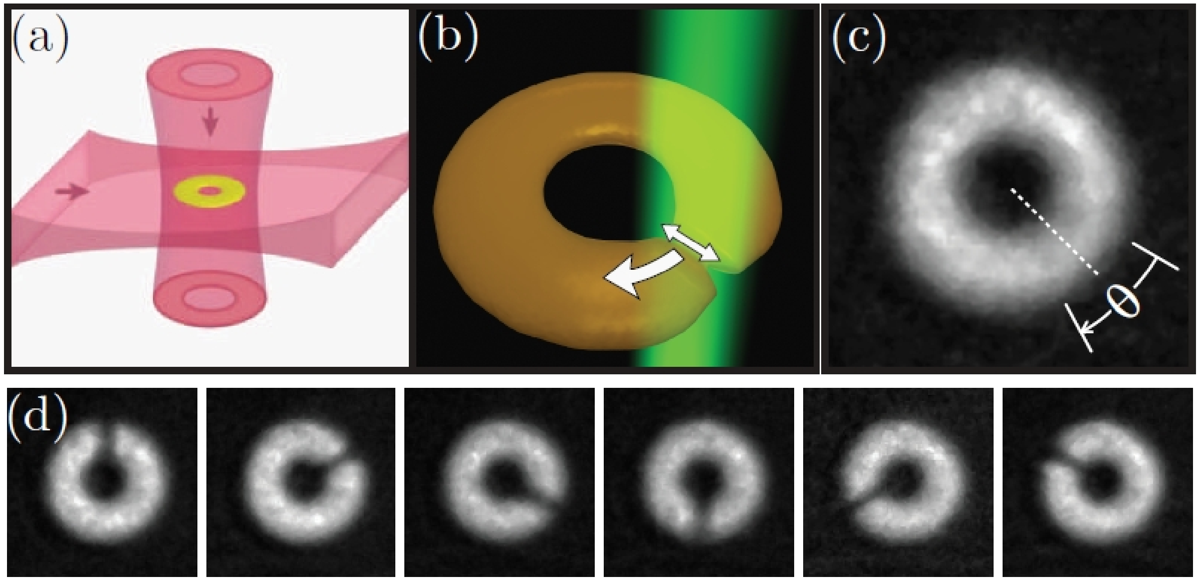
– Friedel oscillations of a Fermi gas near an impurity

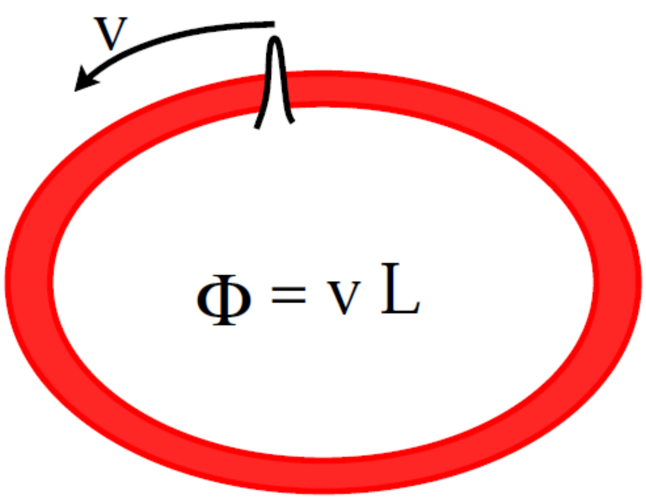


– a moving barrier drags the fluid

– in an infinite, interacting system, the barrier may be very important or negligible depending on interaction strength *[Kane and Fisher]* → *barrier renormalization*

Strongly interacting bosons on a ring under a gauge field (*stirring*)





- Rotation : an artificial gauge field for neutral atoms

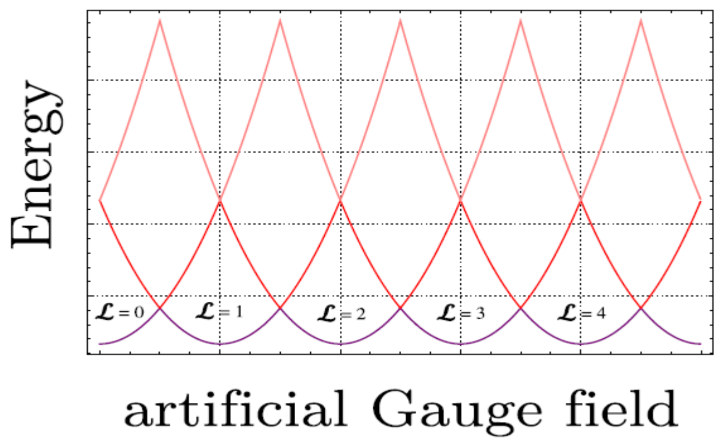
$$\mathcal{H} = \frac{1}{2m}(p - A)^2 + V_{ext} + U_{int}$$

Persistent currents

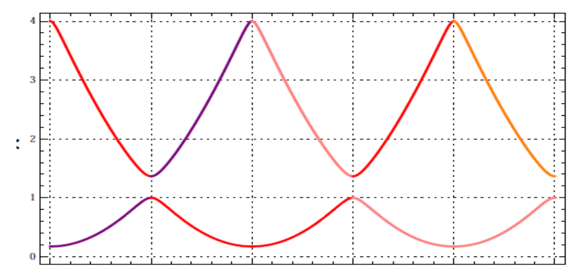
- Ground state energy in presence of gauge field ?

  → periodicity of energy levels

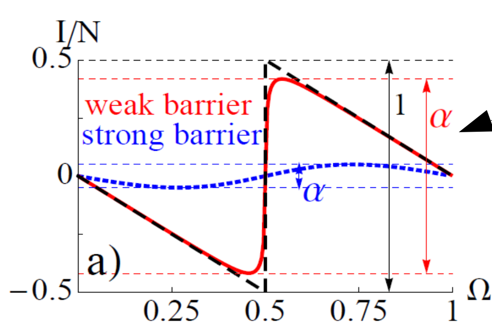
no barrier : Leggett's theorem



with barrier : coherent mixing of angular momentum states

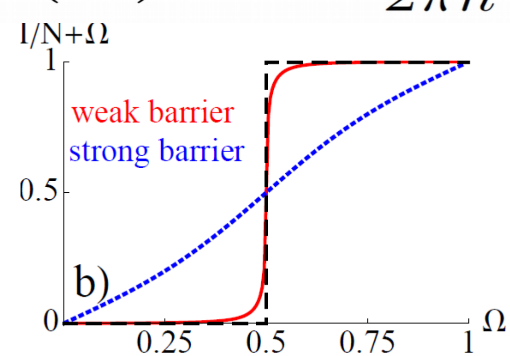


- Persistent currents : definition $$I(\Omega) = -\frac{1}{2\pi\hbar}\frac{\partial E_0(\Omega)}{\partial\Omega}$$



moving frame

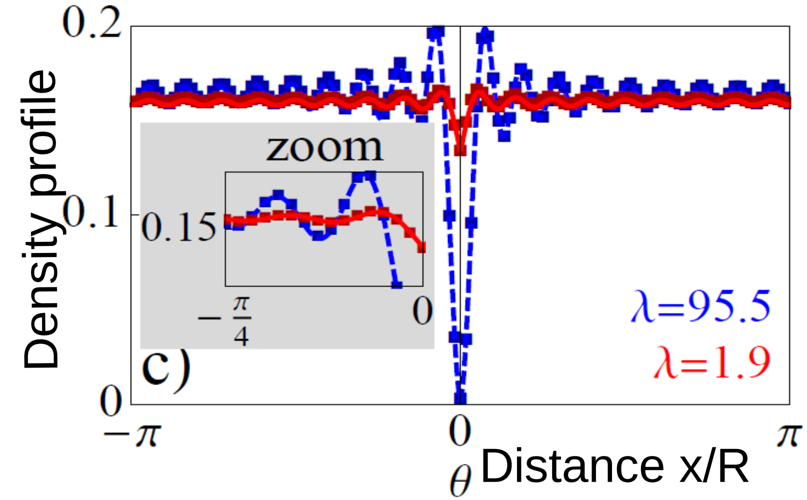
lab frame



Exact results at zero and infinite interactions

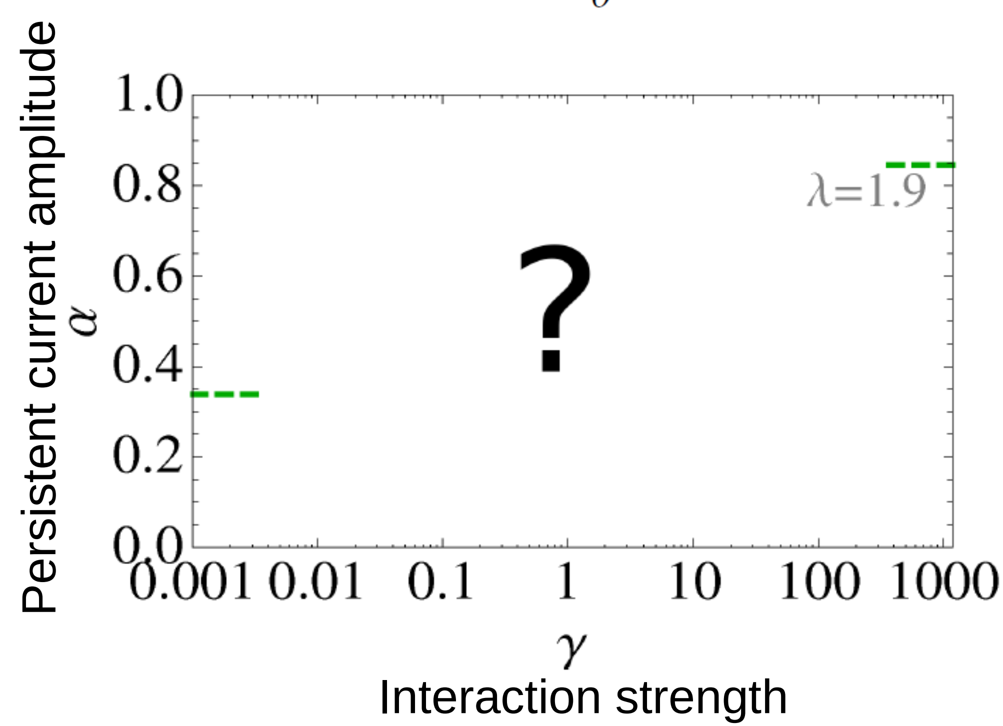
- Density profiles along the ring : Friedel oscillations at strong interactions

  a signature of the strongly correlated regime



- Amplitude of persistent currents : at large interactions, effective barrier

$$U_{eff} = U_0/N$$



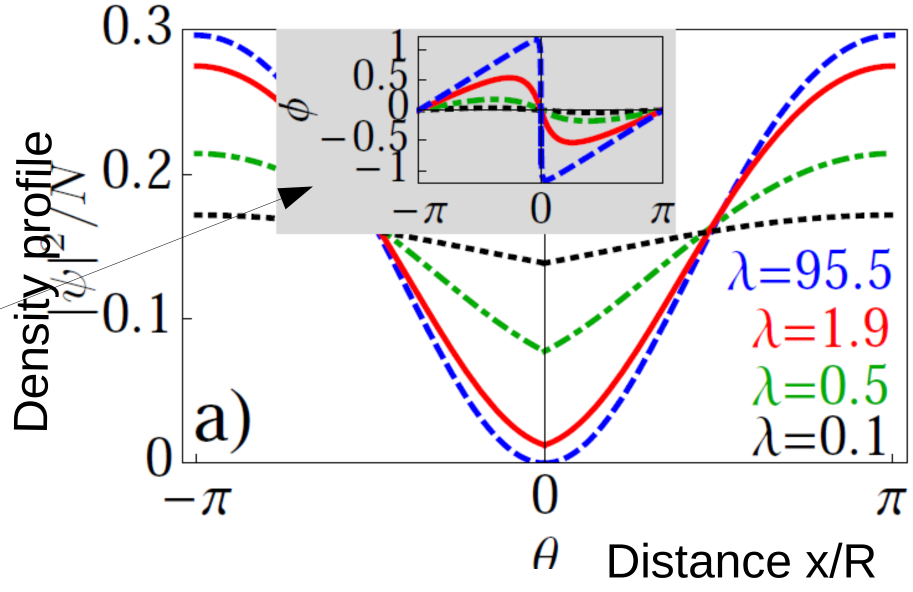
Weakly interacting limit

- Neglect quantum fluctuations : Gross-Pitaevskii equation

$$\frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial x}+\frac{2\pi}{L}\Omega\right)^2\Phi+U_0\delta(x)\Phi+g|\Phi|^2\Phi=\mu\Phi$$

(*new soliton solution in terms of Jacobi elliptic functions)*

- The soliton is pinned by the barrier → ground state
- Phase slips at the position of the barrier



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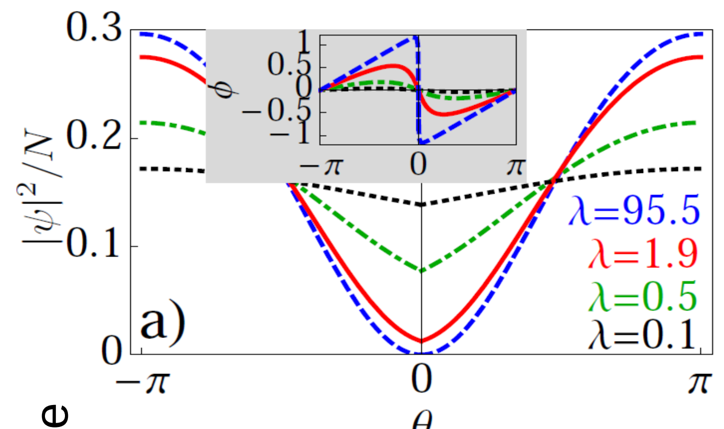

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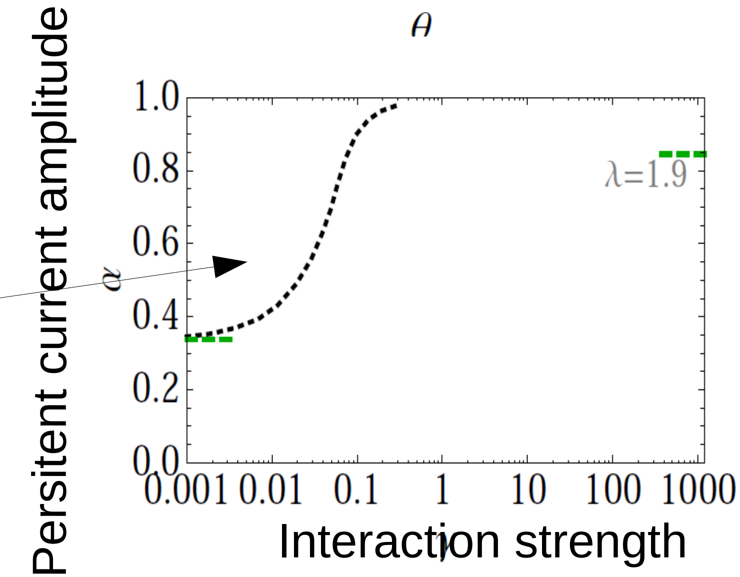

Classical screening of the barrier (too large??)

Strongly interacting limit

- Luttinger liquid theory : quantum fluctuations renormalize the barrier strength $U_{\rm eff} = U_0 (d/L)^K$
- At increasing interactions, density fluctuations renormalize the barrier less and less
  – *(duality) : phase fluctuations are more and more important at increasing interactions*

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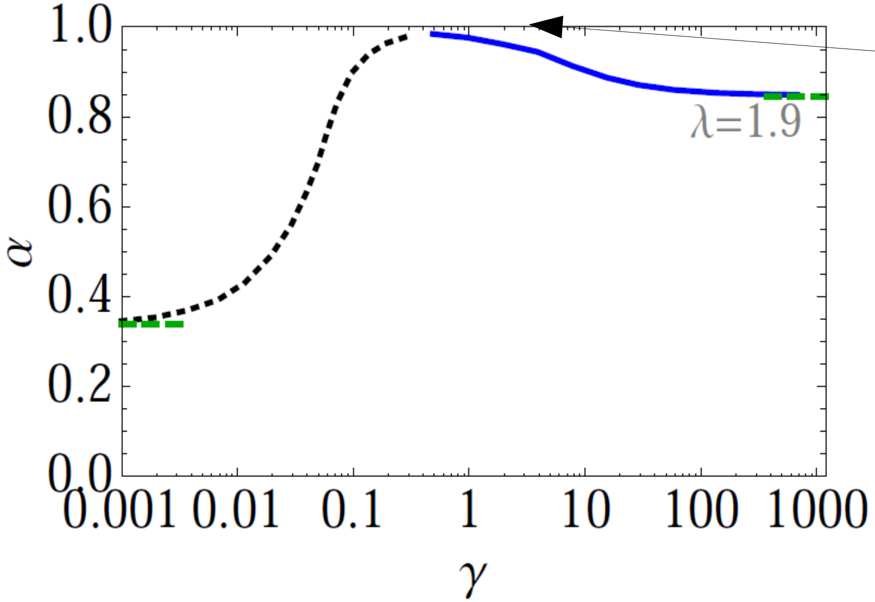
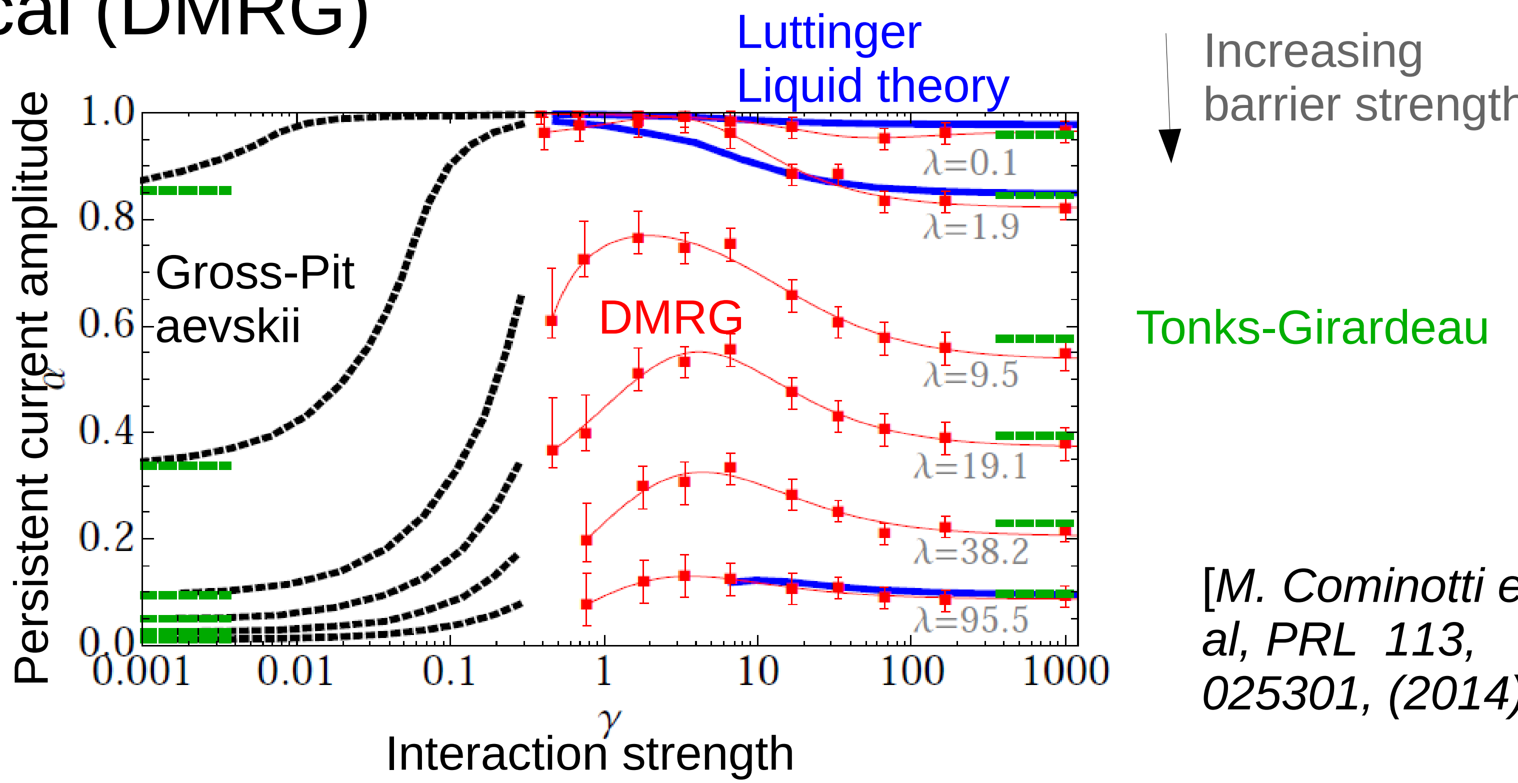


Optimal persistent current at intermediate interactions :

competition of classical screening and quantum fluctuations

Arbitrary interactions and barrier strengths

- Persistent current amplitude : all the analytical results + numerical (DMRG)



[*M. Cominotti et al, PRL 113, 025301, (2014)*]

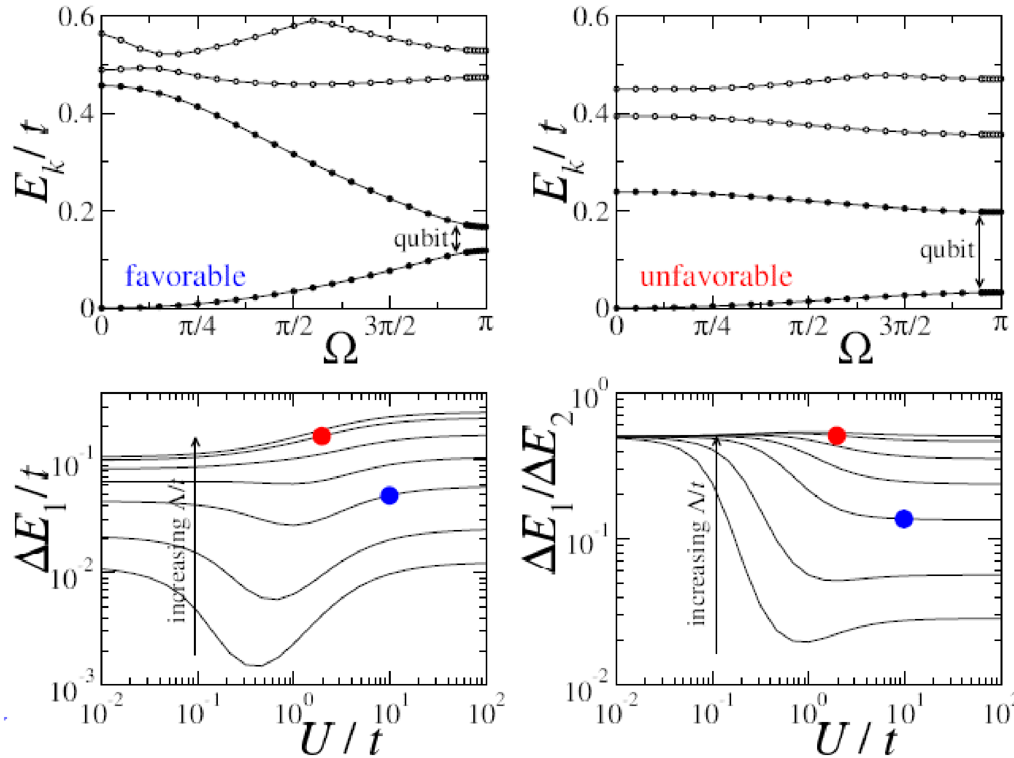
- *Interactions can turn a strong barrier into a weak one* → quantum state manipulation, transport across a barrier, analog of Hawking effect, ...

Excited states of the many-body problem

- From exact diagonalization of the equivalent lattice problem

$$\mathcal{H}_{\rm lat} = -t_{\rm BH} \sum_{j=1}^{N_s} \left(e^{2\pi i \Omega / N_s} b_j^\dagger b_{j+1} + \text{H.c.} \right) + \frac{U_{\rm BH}}{2} n_j (n_j - 1) - \mu_j n_j$$

Eigenvalues vs Coriolis flux (small barrier)



Eigenvalues vs Coriolis flux (large barrier)

Energy difference between first excited and ground state

Ratio of energy gaps

[D. Aghamalyan, *M. Cominotti et al, NJP (2015)]*

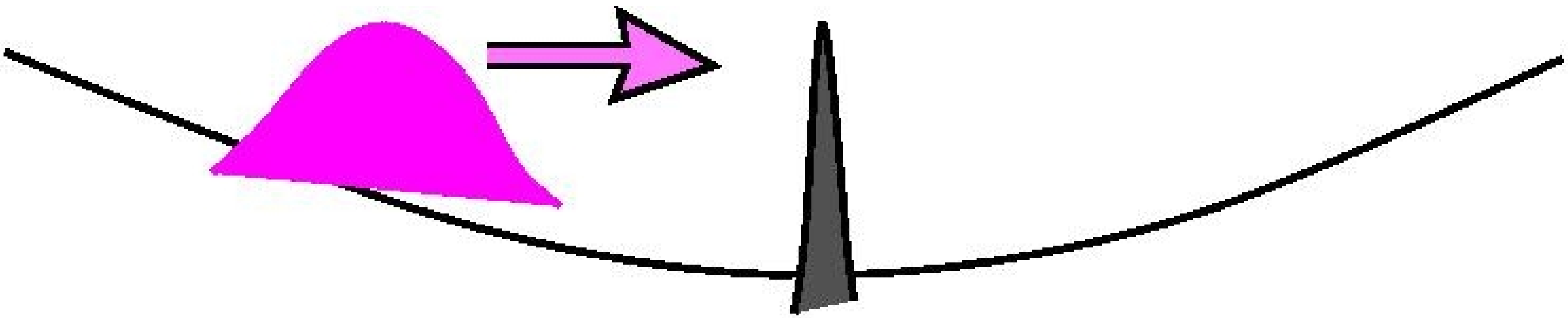
- *A new type of qubit* → macroscopic superposition of current states

Transport across a barrier in interacting quantum gases

→ Finite, inhomogeneous systems ←

→ Arbitrary interactions ←

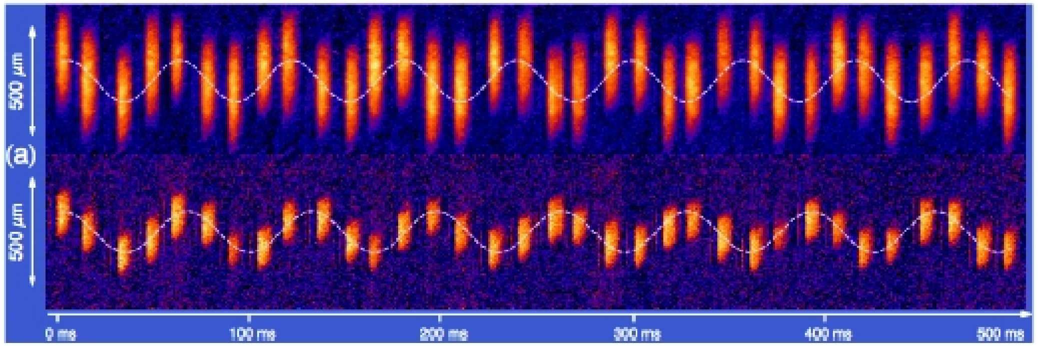
Our idea *: sloshing dipole mode to probe barrier healing and renormalization in a confined geometry*



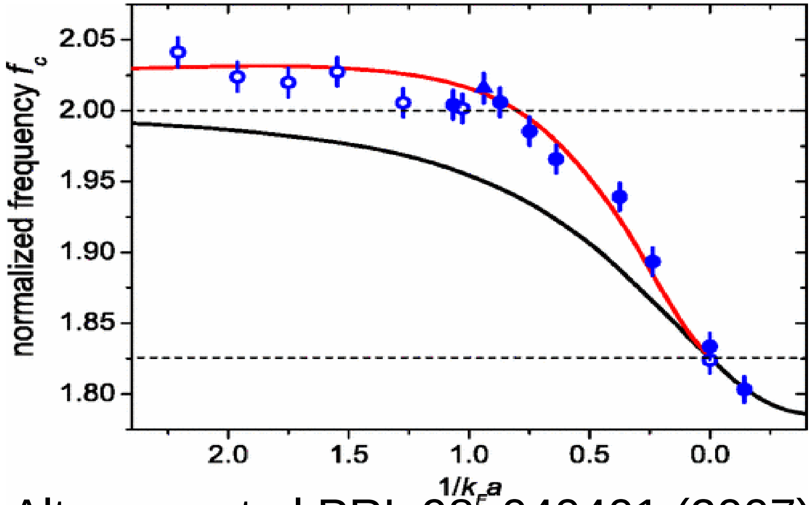
Collective excitations in quantum gases : high-precision tool

Collective-modes frequencies are measured with high precision → information on

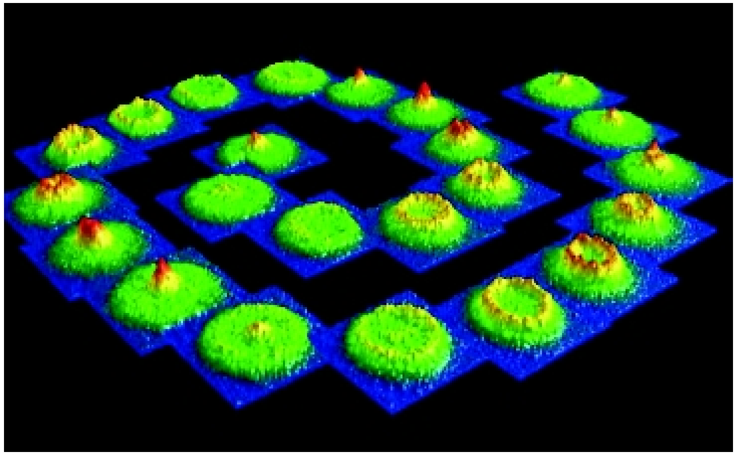
– equation of state
– superfluidity, vortices
– scale invariance
– beyond mean-field effects



Ferrier-Barbut et al Science 345, 1035 (2014)



Altmeyer et al PRL 98, 040401 (2007)



Chevy et al PRL 88, 250402 (2002)

Kohn's theorem for the dipole mode

- *In a purely harmonic trap* $V(x) = \frac{1}{2} m \omega_0^2 x^2$
  the dipole sloshing mode has frequency ω_0

- holds for arbitrary interactions

- not a compressional mode :

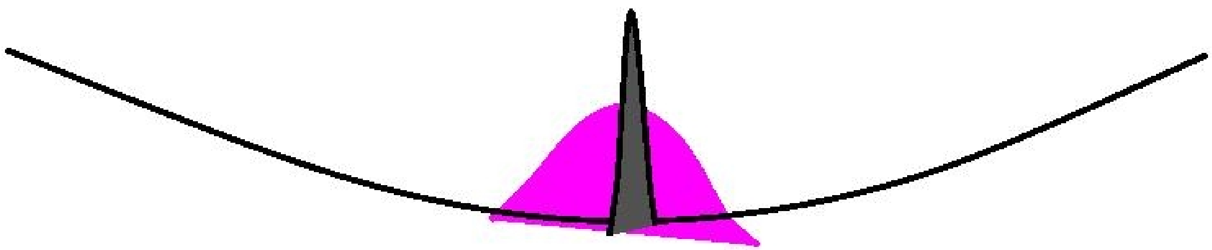
$$n(x,t) = n_0(x - x_0(t))$$

- simmetry preperty, consequence of the harmonic- trap geometry :
  equivalent to looking at the system from an oscillating accelerated frame

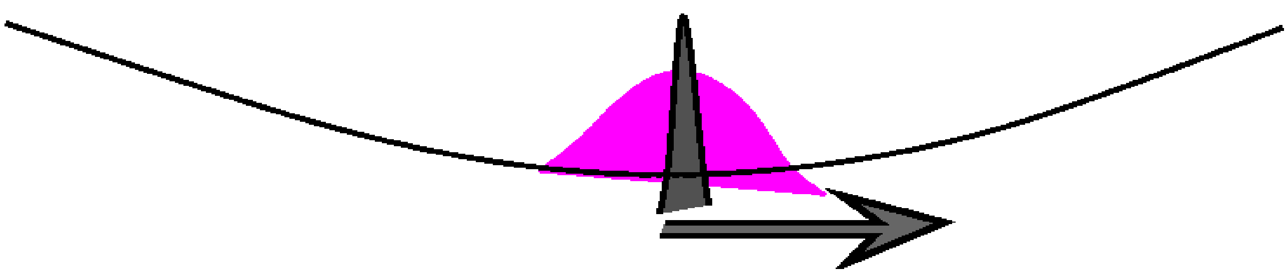
A. Minguzzi PRA 64, 033604 (2001)

Exciting the dipole mode in a split trap : quench protocol

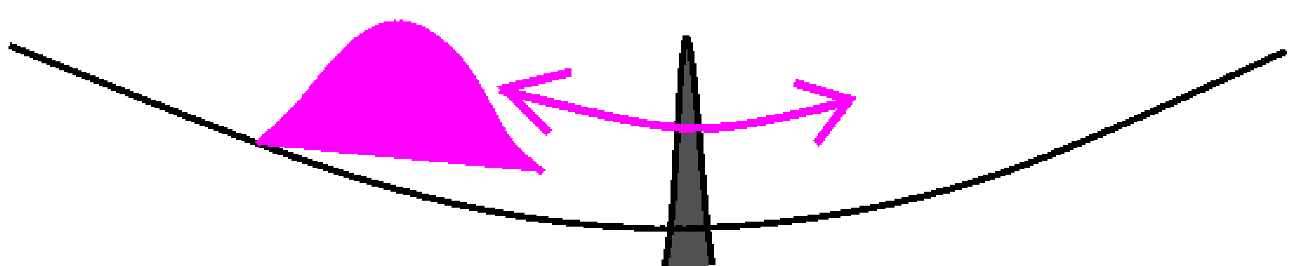
- t<0 : 1D interacting bosons at equilibrium in a *split trap* (harmonic trap + thin barrier)



- t=0 : sudden shift of the split-trap position



- t>0 : time evolution ?



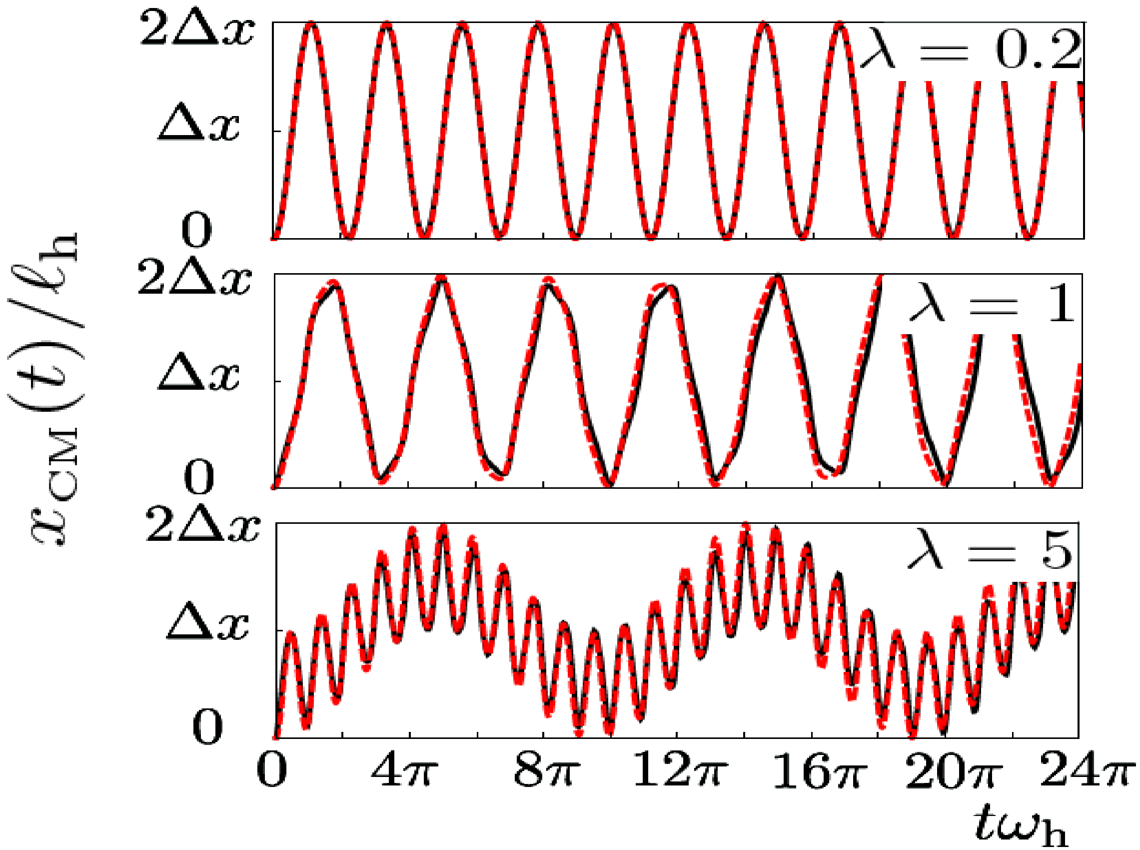
Quench dynamics

for small trap displacement

Follow the center-of mass position $x_{\mathrm{CM}}(t) = \int \mathrm{d}x\, x |n(x,t)|^2$

with $n(x,t) = N \int dx_2 ... dx_N |\Psi(x_1, x_2, ... x_N, t)|^2$

Ideal gas solution



at increasing barrier strength:

→ additional harmonics

→ frequency shift of the dipole mode – *violation of the Kohn's theorem due to the presence of the barrier*

Weak interactions regime

- Gross Pitaevskii equation in a split trap

$$\left[-\frac{\hbar^2}{2m}\partial_x^2 + \lambda\delta(x) + \frac{1}{2}m\omega_{\mathrm{h}}^2 x^2 + gN|\Phi|^2\right]\Phi = \mu\Phi$$

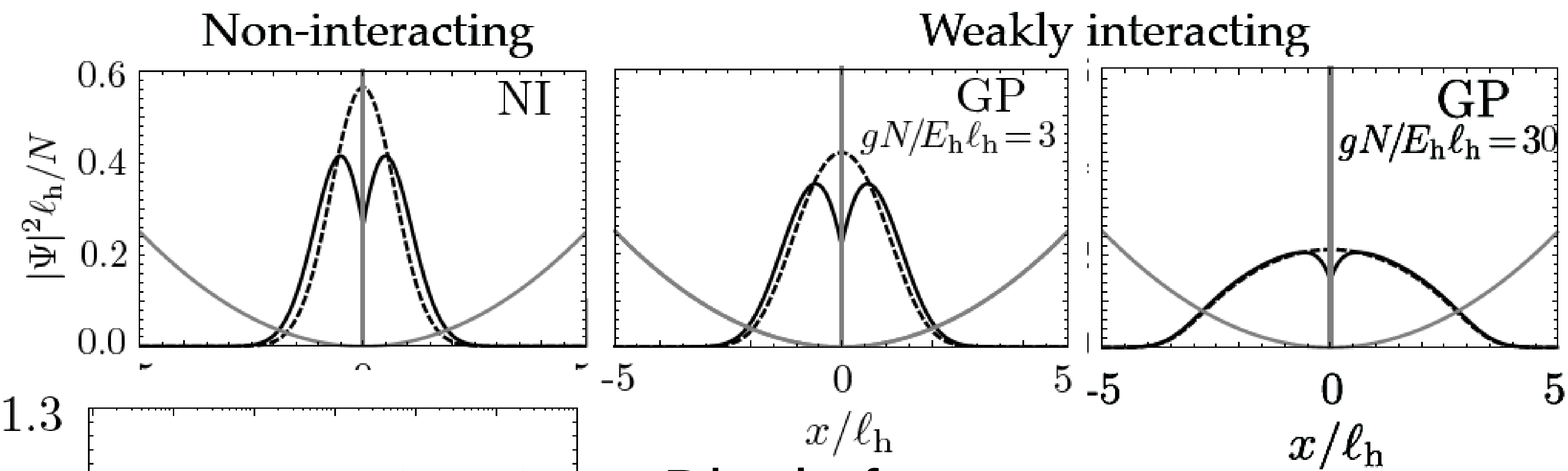
- Initial state : numerical evolution in imaginary times under the pre-quench Hamiltonian

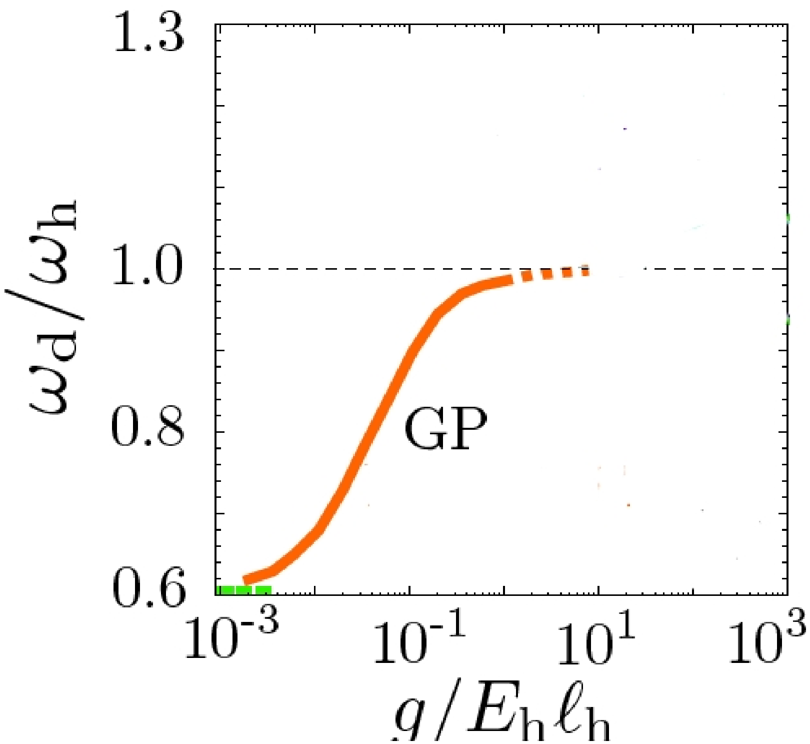
- Dynamics : numerical evolution in real times under the post-quench Hamiltonian (shifted trap)

$$x_{\mathrm{CM}}(t) = \int \mathrm{dx}|\Phi^{\mathrm{t}\geqslant 0}(\mathrm{t},\mathrm{x})|^2\mathrm{x}$$

Weak interactions regime

- Density profiles at increasing interaction strength





- Dipole frequency at increasing interaction strength :

tends to the bare harmonic value due to barrier screening

Infinitely strong interactions : exact solution for the full dynamics

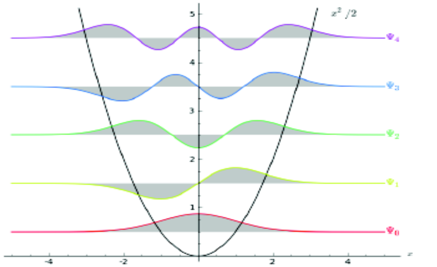
Time-dependent Bose-Fermi mapping *[Girardeau,Wright, PRL (2000)]:*

$$\Psi_B(x_1,...x_N,t) = \Pi_{1\leq j\leq \ell}\mathrm{sgn}(x_j - x_\ell)\Psi_F(x_1,...x_N,t)$$

→ the cusp condition is preserved in the dynamics

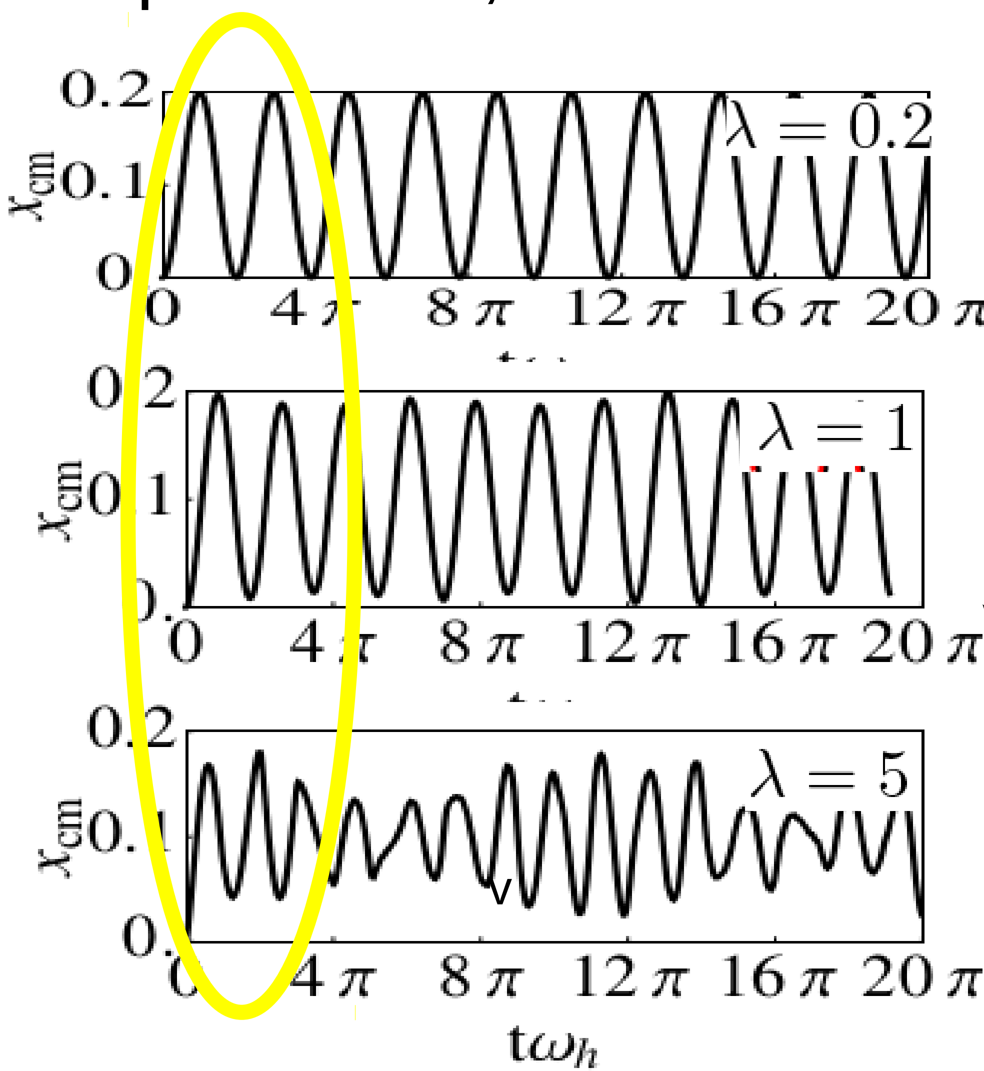
Note : exact solution of the quench dynamics for arbitrary
– barrier strength
– time evolution
– trap shift

Needs the solution of the time-dependent single-particle problem : analytical expression *[Busch et al, J. Phys. B 36, 2553 (2003)]*



Center-of-mass evolution of a Tonks-Girardeau gas

Exact quantum evolution in time – small displacement, various barrier strengths



N=8

the dipolar frequency increases at increasing the barrier strength

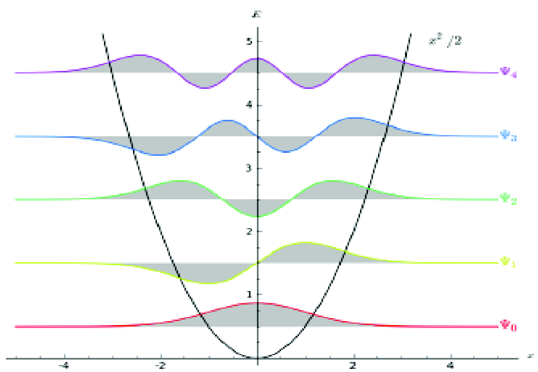
focus on $\lambda = 1$
(small barrier)

Tonks-Girardeau regime :
...prediction of parity effect

Dipole-mode frequency for a weak barrier $U_0\delta(x)$

$$\hbar\omega_{\rm d} = E_1^{\rm TG} - E_0^{\rm TG} = \hbar\omega_{\rm h} + \langle\Psi_1^{\rm TG}|\mathcal{H}_b|\Psi_1^{\rm TG}\rangle - \langle\Psi_0^{\rm TG}|\mathcal{H}_b|\Psi_0^{\rm TG}\rangle$$

$$\Rightarrow \hbar\omega_d = \hbar\omega_{\rm h} + U_0(|\psi_{N+1}(0)|^2 - |\psi_N(0)|^2)$$



for HO confinement, one of the two last orbital vanishes in x=0 → parity effect :

– for N *odd :* $\omega_d < \omega_h$

– for N *even :* $\omega_d > \omega_h$

Tonks-Girardeau regime :
...prediction of parity effect

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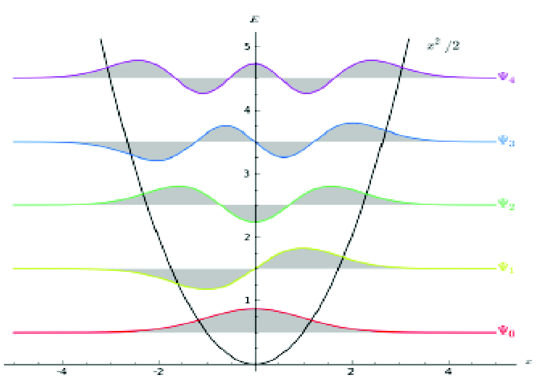

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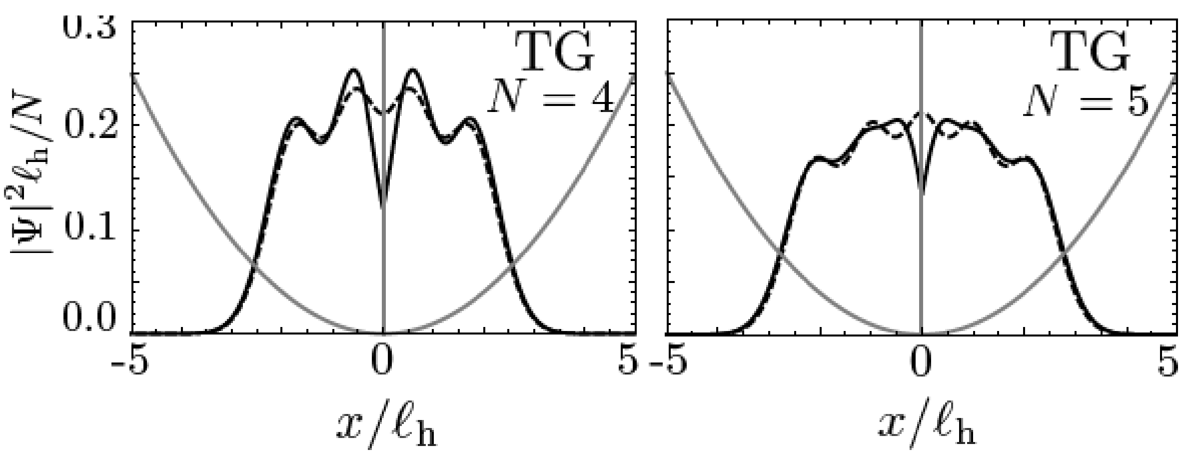

Transport occurs at the Fermi surface for a strongly correlated Bose gas – parity effect as signature of fermionization

Inhomogeneous Luttinger

- Slowly varying inhomogeneity : use the local-density approximation *for the harmonic confinement*

$$\mathcal{H}_0^{\mathrm{LL}} = \frac{\hbar}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}x \left[v_s(x) K(x) (\partial_x \phi(x))^2 + \frac{v_s(x)}{K(x)} (\partial_x \theta(x))^2 \right]$$

- position-dependent Luttinger parameters

$$v_s(x) K(x) = \hbar \pi n(x)/m \qquad \frac{v_s(x)}{K(x)} = \frac{1}{\hbar\pi} \partial_n \mu(n(x))$$

- to proceed analytically, Ansatz for the equation of state

$$\mu(n) = \eta n^{\nu} \quad \begin{cases} \mu(n) = g n & \mathrm{GP} \\ \mu(n) = \dfrac{\hbar^2 \pi^2}{2m} n^2 & \mathrm{TG} \end{cases}$$

$$\nu \leftarrow \text{Bethe ansatz}$$

Normal modes for the inhomogeneous Luttinger liquid

- Mode expansion

$$-\frac{\theta(x,t)}{\pi} = \sum_{j=0}^{\infty} \sqrt{\frac{\hbar n(x)}{2m\omega_j}} \left(\varphi_j(x) e^{i\omega_j t} b_j^\dagger + \varphi_j^*(x) e^{-i\omega_j t} b_j \right)$$

$$\partial_x \phi(x,t) = \sum_{j=0}^{\infty} i \sqrt{\frac{m\omega_j}{2\hbar n(x)}} \left(\varphi_j(x) e^{i\omega_j t} b_j^\dagger - \varphi_j^*(x) e^{-i\omega_j t} b_j \right)$$

- Diagonal Hamiltonian : $\mathcal{H}_0^{\mathrm{LL}} = \sum_{j=0}^{\infty} \hbar\omega_j \left(b_j^\dagger b_j + \frac{1}{2} \right)$
- Mode amplitudes

$$-\omega_j^2 \sqrt{v_s(x) K(x)} \varphi_j(x) = v_s(x) K(x) \partial_x \left(\frac{v_s(x)}{K(x)} \partial_x (\sqrt{v_s(x) K(x)} \varphi_j(x)) \right)$$

- Solution : Gegenbauer polynomials; dispersion :

$$(\omega_j / \omega_{\mathrm{h}})^2 = (j+1)(1 + j\nu/2)$$

Barrier renormalization with Luttinger-Liquid theory

- The barrier is very localized → *cannot be treated with LDA*

$$\mathcal{H}_{\mathrm{b}} = \int_{-\infty}^{\infty} \mathrm{d}x \, U_0 \delta(x) \rho(x)$$

- Integrating out the higher-energy density fluctuation modes :

$$\boxed{\mathcal{H}_{\mathrm{b}}^{LL} \sim 2n(0) U^{\mathrm{eff}} \cos[2\theta_0(0) + 2\pi \int_{-\infty}^{0} \mathrm{d}x \; n(x)]}$$

from the LL expression for the density operator

$$\rho(x) = [n(x) + \partial_x \theta(x)/\pi] \sum_{l=-\infty}^{+\infty} e^{2il\theta(x) + 2il\pi \int_{-\infty}^{x} \mathrm{d}x' n(x')}$$

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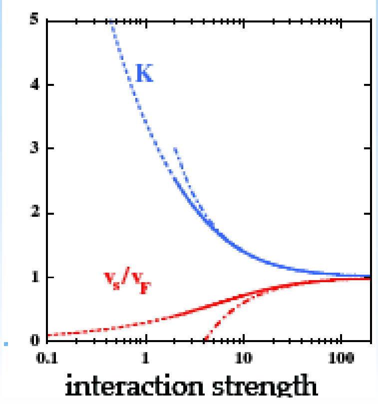
- Barrier renormalization by quantum fluctuations of the density:

$$U^{\mathrm{eff}} = U_0 \langle 0| \cos(2 \sum^{N/c} \theta_j(0)) |0\rangle \sim U_0 \left(\frac{a}{N}\right)^{\kappa}$$

$$\kappa = K(0) v_s(0)/\omega_\mathrm{h} R = K_0 \sqrt{\frac{\nu}{2}}$$

$$\langle \rho(x) \rho(0) \rangle \sim |x|^{-2K}$$



- Ueff decreases at decreasing interactions
- The exponent is *different from the homogeneus case* !

Barrier renormalization with Luttinger-Liquid theory

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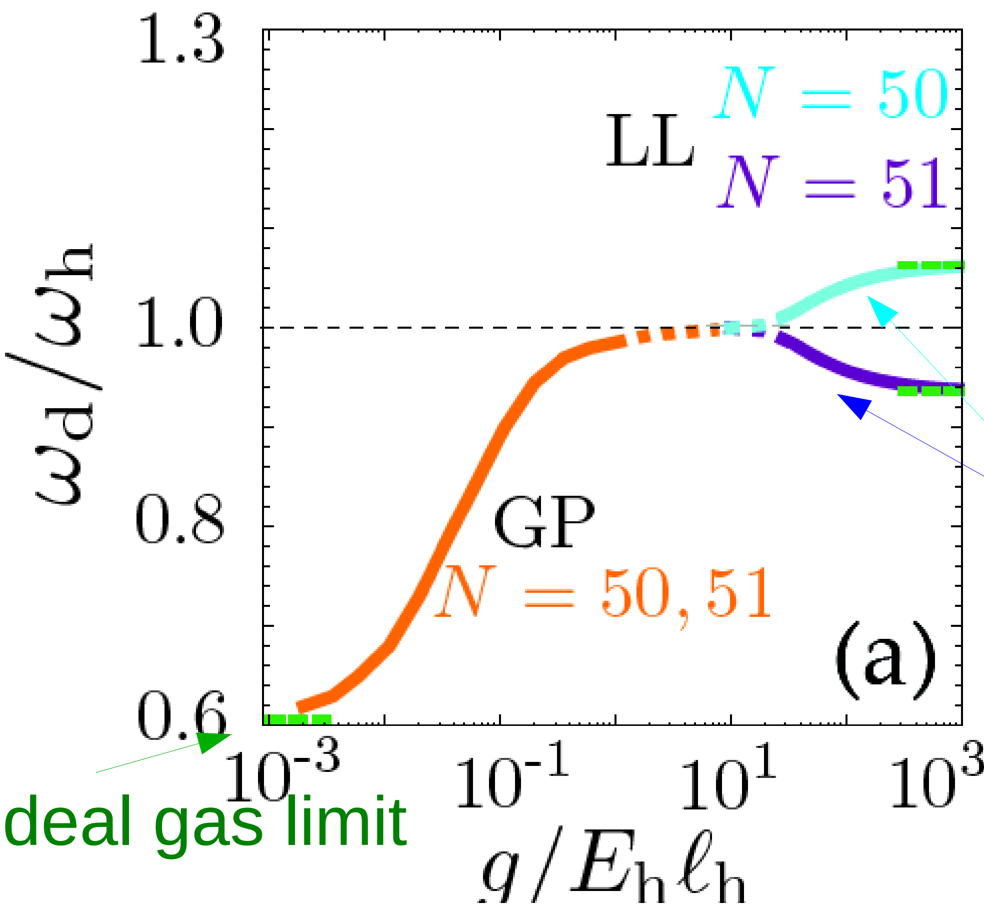
- Parity effect :

$$(-1)^N$$

Dipole mode frequency vs interaction strength

- Main result – from Gross-Pitaevskii, Tonks Girardeau and Luttinger liquid theory



Barrier strength

$\lambda = 1$

Tonks-Girardeau exact result

Phase-density duality : at *increasing* interactions, barrier renormalization *decreases*

$$U^{\text{eff}} = U_0 \left(\frac{a}{N}\right)^{\kappa}$$

Ideal gas limit

The frequency shift is a direct measure of the competition of barrier screening and renormalization by quantum fluctuations

Cominotti, Hekking, Minguzzi, PRA 93, 033628 (2015)

Numerical approach
for small trap displacement

Dipole-mode frequency from perturbation theory :

$$|\Psi_0^{t\geqslant 0}(t)\rangle = \exp(-i\mathcal{H}^{t\geqslant 0}t/\hbar)|\Psi_0^{t<0}\rangle$$
$$\mathcal{H}^{t<0} \simeq \mathcal{H}^{t\geqslant 0} + \Delta x \partial_x V_{\mathrm{ext}}^{t\geqslant 0}$$
$$|\Psi_0^{t<0}\rangle = |\Psi_0^{t\geqslant 0}\rangle + \Delta x \sum_{k=0}^{\infty} \frac{\langle \Psi_k^{t\geqslant 0}|\partial_x V_{\mathrm{ext}}^{t\geqslant 0}|\Psi_0^{t\geqslant 0}\rangle}{(E_0^{t\geqslant 0} - E_k^{t\geqslant 0})}|\Psi_k^{t\geqslant 0}\rangle$$

$$\boxed{\Rightarrow \quad \omega_{\mathrm{d}} = (E_1^{t\geqslant 0} - E_0^{t\geqslant 0})/\hbar}$$

Ground and first-excited state from numerical diagonalization – arbitrary interactions

Exact diagonalization method

Determine to high accuracy the ground- and first- excited state of the many-body Hamiltonian

$$\mathcal{H} = \sum_{j=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2 x_j} + U_0\delta(x_j) + \frac{1}{2}m\omega_{\mathrm{h}}^2 x_j^2 + \frac{g}{2}\sum_{j,l=1}^{N}\delta(x_l - x_j)$$

- represented on the basis of the single particle problem
- truncated Hilbert space :

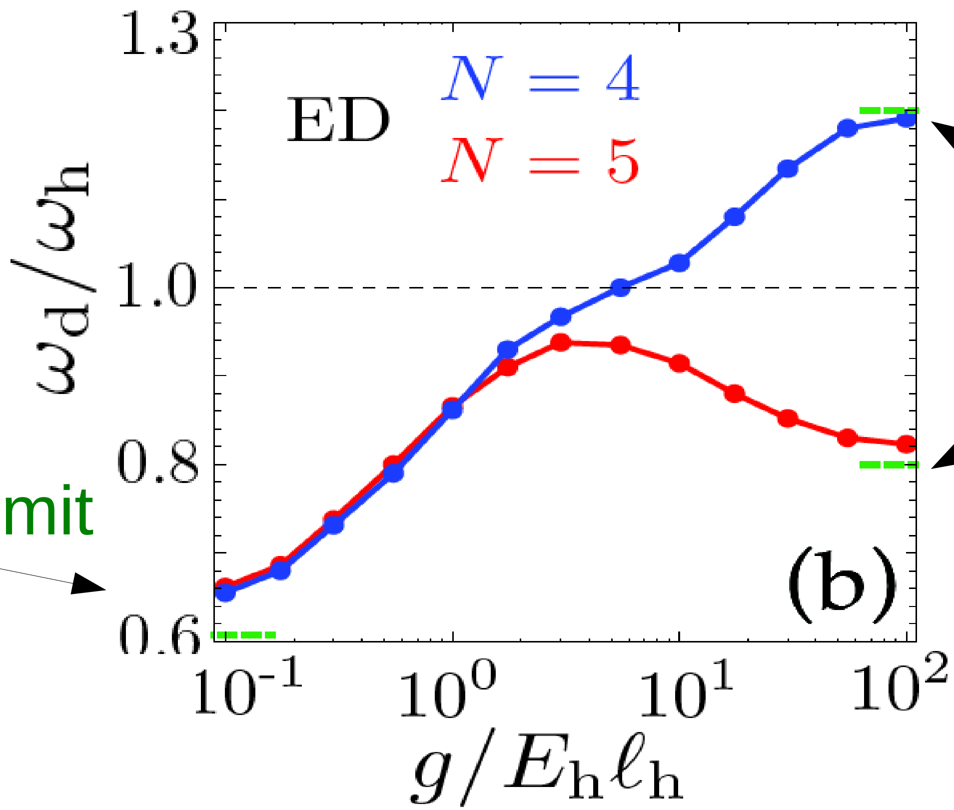
$$\begin{array}{l} N \text{ \# particles} \\ S \text{ \# states} \end{array} \Rightarrow \begin{array}{c} \text{Size} \\ \text{Hilbert} \\ \text{space} \end{array} \quad \binom{S+N-1}{N}$$

Number of particles limited to N< 10

$\Rightarrow$ Importance of Luttinger Liquid theory to treat larger N – need of benchmark

Dipole mode frequency vs interaction strength

Exact diagonalization results for N=4 and 5



Barrier strength $\lambda = 1$

Tonks-Girardeau exact result

Ideal gas limit

Cominotti, Hekking, Minguzzi, PRA 93, 033628 (2015)

- Parity effect at large interactions : an effect of correlations
- Nontrivial frequency shift with interaction strength ...

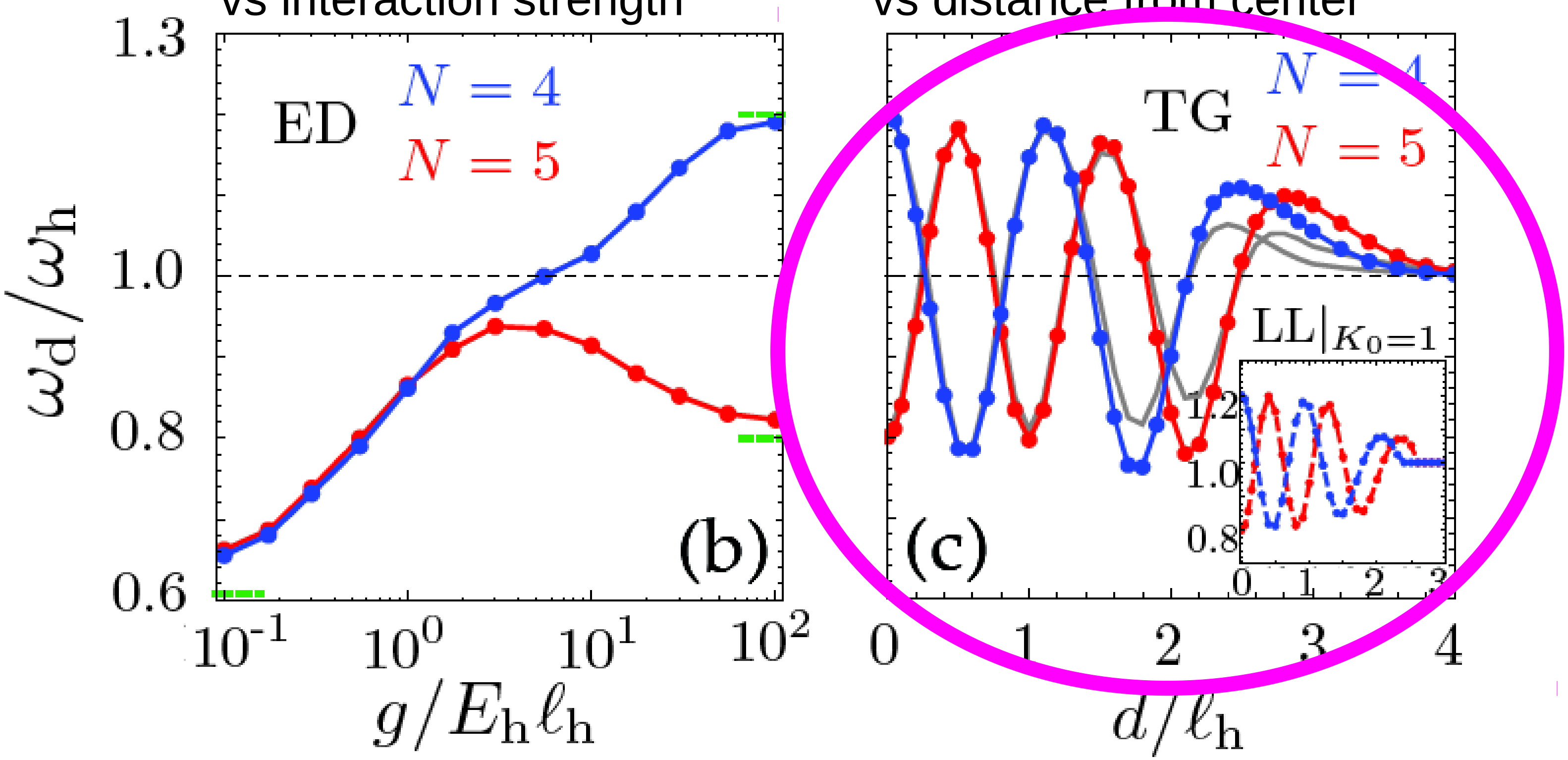
→ barrier screening and renormalization

Effect of non-centered barrier

- Dipole-mode frequency from Tonks-Girardeau and Luttinger liquid theory

centered barrier, frequency vs interaction strength

non-centered barrier, frequency vs distance from center

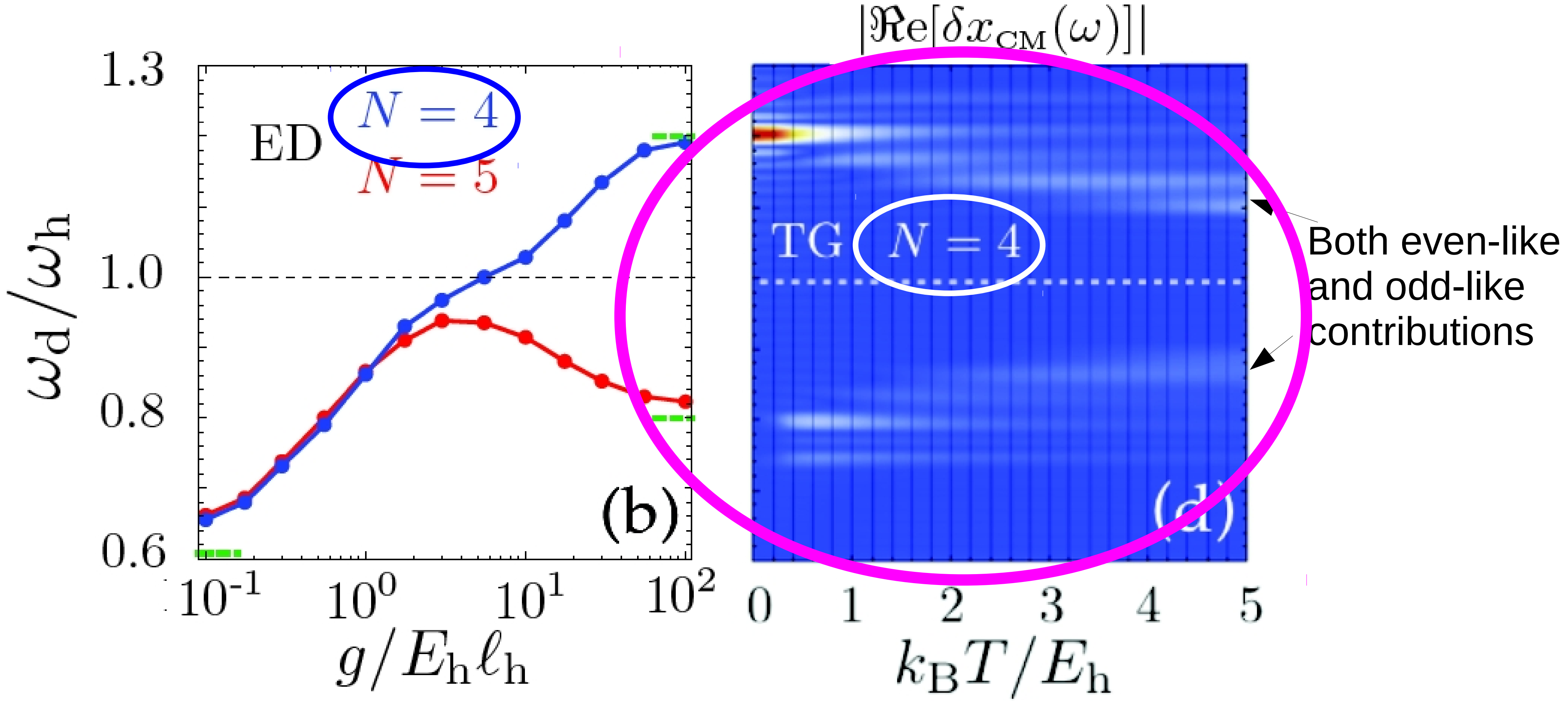


Oscillatory behaviour vs barrier distance, 'particle-counting effect' – well accounted for by the Luttinger-liquid theory

Cominotti, Hekking, Minguzzi, PRA 93, 033628 (2015)

...and at finite temperature ?

- Exact solution for the quench dynamics at finite temperature using the thermal Bose-Fermi mapping
- Dipole frequency from a Fourier analysis of $x_{\rm CM}(t)$

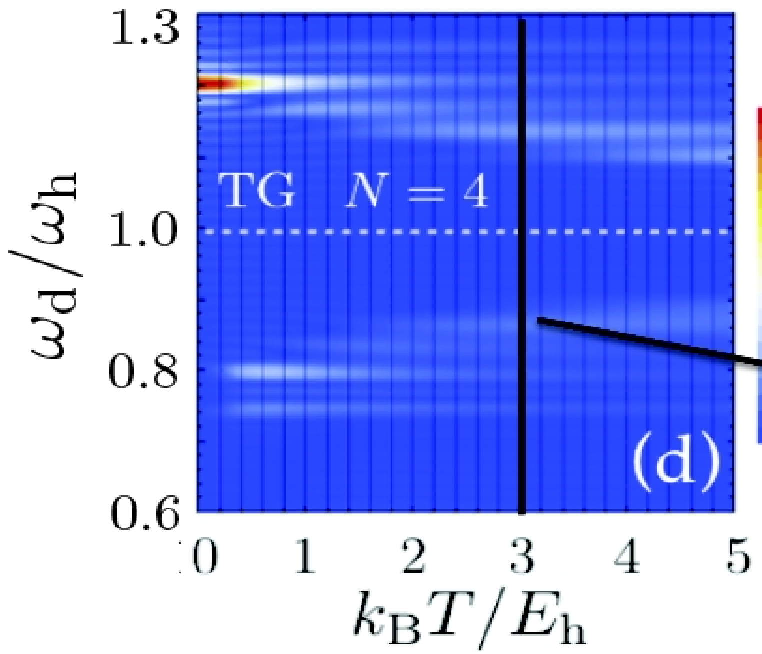


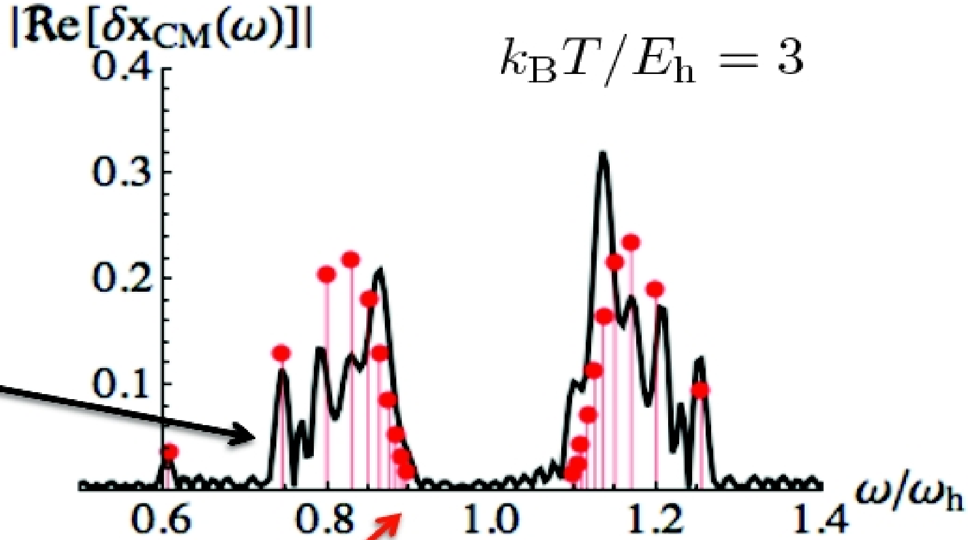
Cominotti, Hekking, Minguzzi, PRA 93, 033628 (2015)

Spectral function at finite T

- Understanding the frequency contributions to the center-of-mass motion at finite temperature

$|\Re\mathrm{e}[\delta x_{\mathrm{CM}}(\omega)]|$ from exact dynamical evolution (on a finite time)





Linear response theory:

$$V_{\mathrm{p}}(x,t) = \theta(t)\Delta x(m\omega_{\mathrm{h}}^2 x + U_0\delta'(x)) \qquad \delta x_{\mathrm{CM}}(\omega) = \int \mathrm{d}x\, x \int \mathrm{d}x'\, \chi(x,x';\omega)V_{\mathrm{p}}(x',\omega)$$

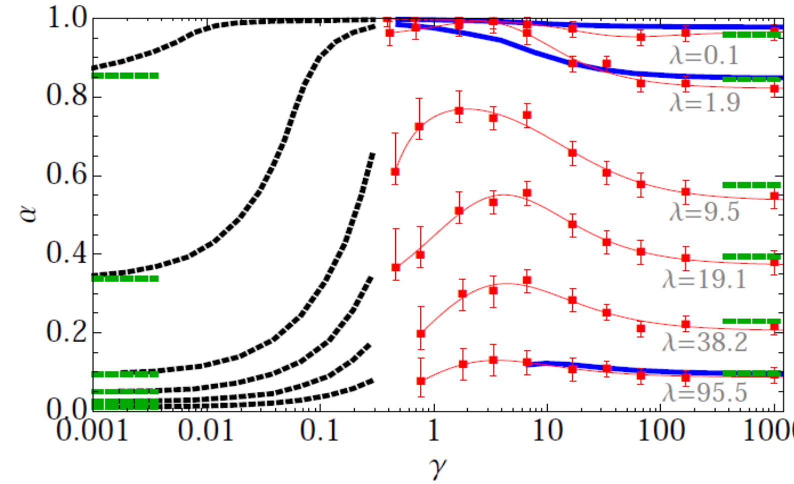
$$\chi(x,x';\omega) = (1/\hbar)\sum_{j\neq k}\psi_j^*(x)\psi_k(x)\psi_k^*(x')\psi_j(x')f(\varepsilon_j)[1-f(\varepsilon_k)]$$

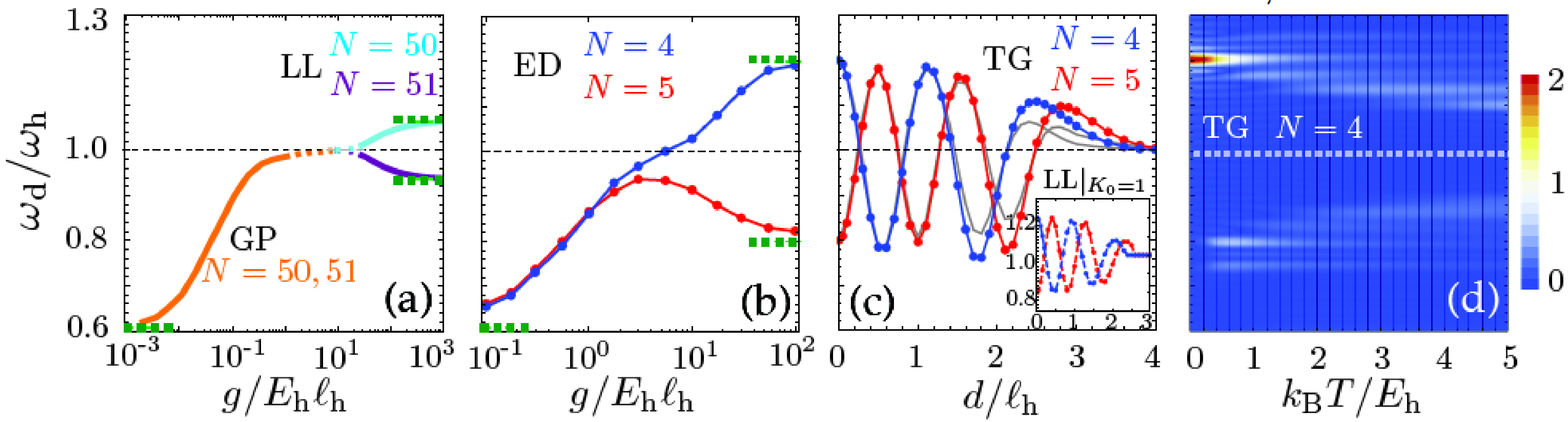
$$\times\left(\frac{1}{(\omega-(\varepsilon_k-\varepsilon_j)/\hbar))+i\epsilon} - \frac{1}{(\omega+(\varepsilon_k-\varepsilon_j)/\hbar))+i\epsilon}\right)$$

Parity effect still visible at finite temperature !

Conclusions

- Persistent currents and dipole modes as a powerful tool to explore effects of quantum fluctuations





- Competition between barrier screening at weak interactions and renormalization at strong interactions

- Parity effect due to onset of strongly correlated regime & fermionization

Outlook

- Persistent currents : beyond the strictly 1D regime, link to experiments
- Dipole oscillations : beyond the small-shift regime
  - damping ?
  - dynamics at long times ?
- Extensions : fermions, dipolar gases, multicomponents,...

Many thanks to...

- Frank Hekking, Marco Cominotti (LPMMC, Grenoble)
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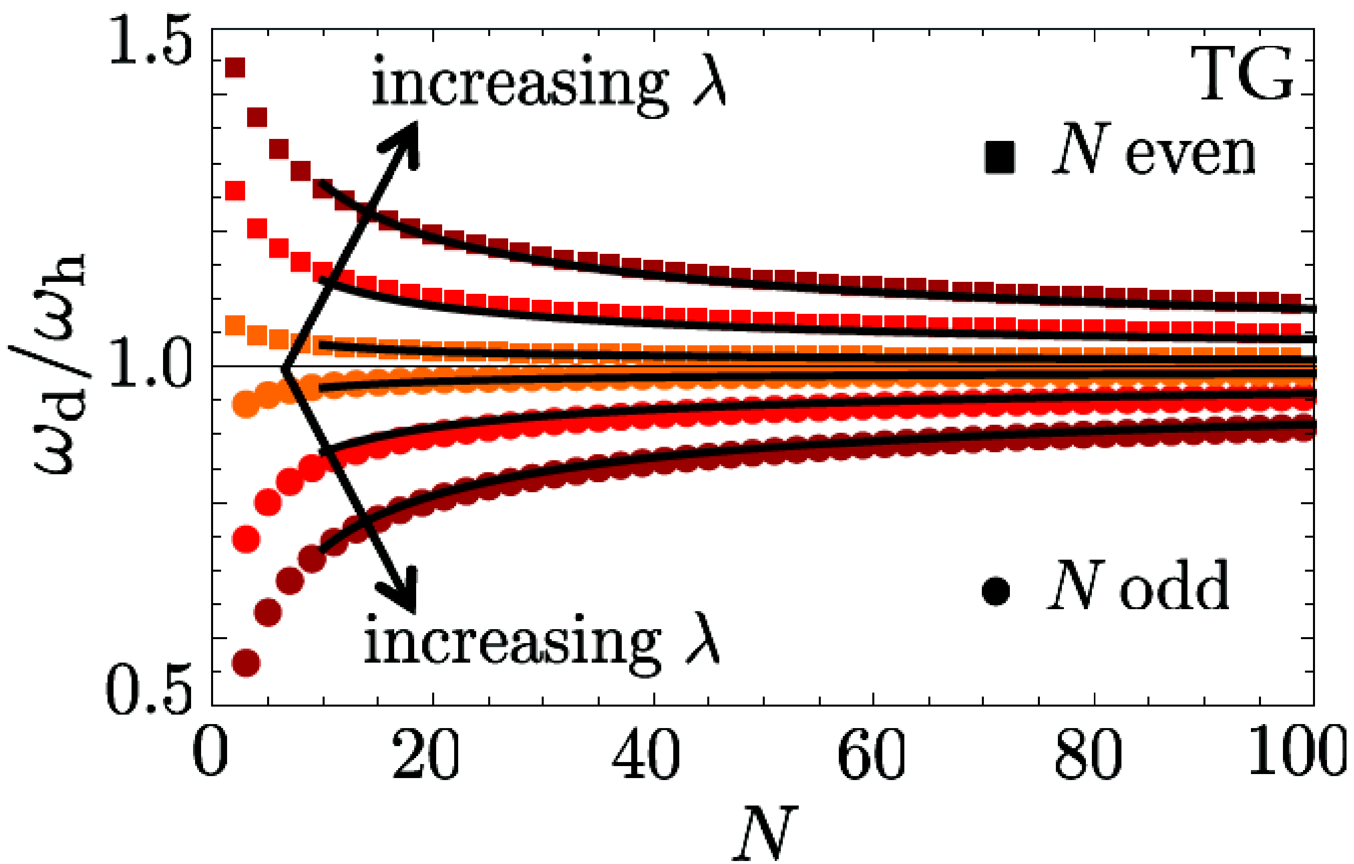
- Luigi Amico (Catania and Singapure)
- Davit Agamalyan, Leong Kwek (Singapure)

Scaling of parity gap with size

- The parity effect vanishes in the thermodynamic limit

- From the Tonks-Girardeau solution :

$$|\omega_{\rm d} - \omega_{\rm h}| \propto 1/\sqrt{N}$$



Effective barrier : similar results at larger N with a larger barrier